Research Perspectives

Trigonometry Behind The House Cusps

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HISTORY:

When the author was studying astrology course, he has given lecture on astronomy and basics of the celestial mechanics in the Asian astrologer's Meet 1999 held at Chennai, organized by the KP Stellar Astrological Research Institute (KPSARI) Chennai, INDIA. During the time of lecture the author noticed that the most of astrologers are more focused on the analysis and prediction part and they are least interested in astronomy, mathematical part except very few were showing interest. Even in the recent days the situation is same.

But one thing we should remember that for all our analysis and prediction there is a main part that is the mathematics that are used for the computation of house cusps & planetary positions etc are very much required. To simplify our work nowadays there are many ready reckoners and computer software are available which minimizes our work considerably. So obviously our knowledge growth is very limited in astronomy and the mathematics used behind the astrology.

For astronomy there are many books and on-line resources are available that may be used to study about the subject. But while studying the on-line resources the author found that the some of the web sites are showing the wrong equations and even the articles presented online also with some mistakes. The author was very curious to know the derivation of the equations and in this regard asked the authors of the respective website/Article/Discussion group members to provide the basic diagrams used for the formulation of the equations. But unfortunately no one has given the exact or even similar details about describing the use of spherical trigonometry and formulation of the equations with proper spherical triangle diagram to support the formulation and the response was not up to the satisfactory level.

SCOPE:

In the KP System of Astrology, Krishnamurti Paddhati(KP) astrologers are using the Placidus House system for calculating the house cusp positions. So in this article, the basics of the astronomy and the use of spherical trigonometry for the formulation of the Placidus house cusp equations with proper spherical triangle diagram to support the equations/formulas are presented to give mathematical awareness to the astrologers. This article is purely intended to show the math parts that are used and not to develop either any ready reckoners/computer software or to verify any available data.

PLANE TRIGONOMETRY:

Plane trigonometry describes the triangles on the plane and the relationships between the sides and the angles. This is very much used for calculations in various fields like mathematics, physics, Engineering, astronomy and earth-surface, orbital and space navigation etc. There are some basic plane trigonometric equations are used for the formulation of the house cusp equations. Even though it is available in the mathematics books and online websites it is provided here for the reader's ready reference and use.

Right Angle Triangle

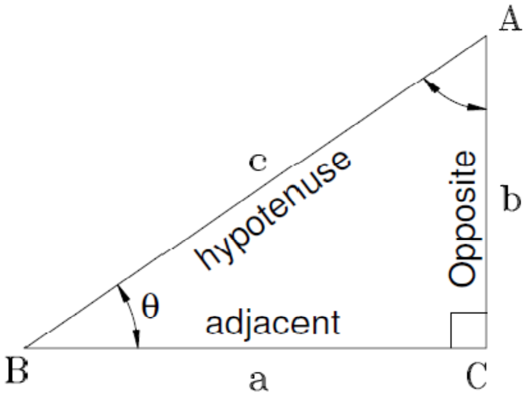

Fig.1) Right-Angled Triangle

In the right angled triangle ABC with right angle C = $\pi/2$, A + B = $\pi/2$

$$\sin\theta = \frac{b}{c} = \frac{opposite}{hypotenuse} \qquad \csc\theta = \frac{c}{b} = \frac{hypotenuse}{opposite}$$

$$\cos\theta = \frac{a}{c} = \frac{adjacent}{hypotenuse} \qquad \sec\theta = \frac{c}{a} = \frac{hypotenuse}{adjacent}$$

$$\tan\theta = \frac{b}{a} = \frac{opposite}{adjacent} \qquad \cot\theta = \frac{a}{b} = \frac{adjacent}{opposite}$$

Reciprocal & Quotient Identities

$$\csc\theta = \frac{1}{\sin\theta} \quad \sec\theta = \frac{1}{\cos\theta} \quad \cot\theta = \frac{1}{\tan\theta} \quad \tan\theta = \frac{\sin\theta}{\cos\theta} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sec^2\theta - \tan^2\theta = 1$$
$$\csc^2\theta - \cot^2\theta = 1$$

Even-Odd & Co-Function Identities (Where θ is in Degrees)

$$\sin(-\theta) = -\sin\theta \qquad \sin(90-\theta) = +\cos\theta \qquad \sin(180-\theta) = +\sin\theta$$
$$\cos(-\theta) = +\cos\theta \qquad \cos(90-\theta) = +\sin\theta \qquad \cos(180-\theta) = -\cos\theta$$
$$\tan(-\theta) = -\tan\theta \qquad \tan(90-\theta) = +\cot\theta \qquad \tan(180-\theta) = -\tan\theta$$
$$\csc(-\theta) = -\csc\theta \qquad \csc(90-\theta) = +\sec\theta \qquad \csc(180-\theta) = +\csc\theta$$
$$\sec(-\theta) = +\sec\theta \qquad \sec(90-\theta) = +\csc\theta \qquad \sec(180-\theta) = -\sec\theta$$
$$\cot(-\theta) = -\cot\theta \qquad \cot(90-\theta) = +\tan\theta \qquad \cot(180-\theta) = -\cot\theta$$

$$
\begin{array}{lll}
\sin(90+\theta)=+\cos\theta & \sin(180+\theta)=-\sin\theta & \sin(360+\theta)=+\sin\theta \\
\cos(90+\theta)=-\sin\theta & \cos(180+\theta)=-\cos\theta & \cos(360+\theta)=+\cos\theta \\
\tan(90+\theta)=-\cot\theta & \tan(180+\theta)=+\tan\theta & \tan(360+\theta)=+\tan\theta \\
\csc(90+\theta)=+\sec\theta & \csc(180+\theta)=-\csc\theta & \csc(360+\theta)=+\csc\theta \\
\sec(90+\theta)=-\csc\theta & \sec(180+\theta)=-\sec\theta & \sec(360+\theta)=+\sec\theta \\
\cot(90+\theta)=-\tan\theta & \cot(180+\theta)=+\cot\theta & \cot(360+\theta)=+\cot\theta
\end{array}
$$

The Addition Formulas

$$\sin(A+B)=\sin A\cos B+\cos A\sin B$$

$$\sin(A-B)=\sin A\cos B-\cos A\sin B$$

$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$

$$\cos(A-B)=\cos A\cos B+\sin A\sin B$$

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$$

Napier's Rules

Sine Rule	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
Cosine Rule (Side)	$c^2=a^2+b^2-2ab\cos C$
Cosine Rule (Angle)	$\cos C=\frac{a^2+b^2-c^2}{2ab}$
Tangent Rule	$\frac{a-b}{a+b}=\frac{\tan\left[\frac{1}{2}(A-B)\right]}{\tan\left[\frac{1}{2}(A+B)\right]}$

SPHERICAL TRIGONOMETRY:

Spherical trigonometry describes the triangles on the sphere and the relationships between the sides and the angles. This is very much used for calculations in astronomy and earth-surface, orbital and space navigation.

Spherical Triangle

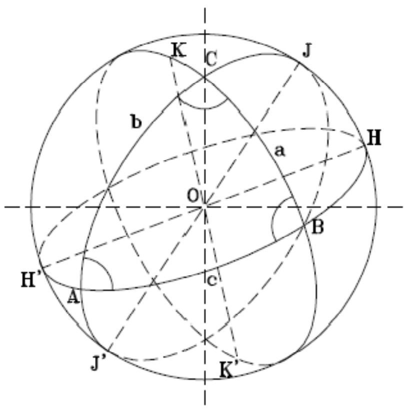

Fig.2) Sphere & Spherical Triangle

In figure 2, Let H'J'HJ be the sphere whose centre is at O. Also let J'ACJ, H'ABH & K'BCK are the great circles of the sphere whose centre is at O and intersecting the surface of the sphere at the points A, B, C. The part of the sphere surface subtended at these three points is called Spherical triangle ABC. Similar to plane triangles, we denote the three spherical angles by A, B, C and the sides opposite to them by arc BC = a, arc AC = b, arc AB = c. Before we go into the formulas some important points given below may be noted.

1) The sum of the three angles of a spherical triangle is always greater than 180. That is the Spherical Excess denoted by E = A + B + C > π
2) The formulas are valid only for triangles in which the three sides are arcs of great circles.
3) The sides of a spherical triangle, as well as the angles, are all expressed in angular measure (degrees, minutes and seconds).

Napier's Rules for Spherical Triangle

Rule	Equations
Sine Rule	$\frac{\sin a}{\sin A}=\frac{\sin b}{\sin B}=\frac{\sin c}{\sin C}$
Cosine Rule (Side)	$\cos c=\cos a\cos b+\sin a\sin b\cos C$
Cosine Rule (Angle)	$\cos A=-\cos B\cos C+\sin B\sin C\cos a$
Cotangent Rule	$\cot a\sin b=\cot A\sin C+\cos b\cos C$
Cosine Rule (Except 1 Angle)	$\sin c\cos A=\cos a\sin b-\sin a\cos b\cos C$
Cosine Rule (Except 1 Side)	$\sin C\cos a=\cos A\sin B+\sin A\cos B\cos c$
Tanget Rule	$\frac{\tan\left[\frac{1}{2}(B-C)\right]}{\tan\left[\frac{1}{2}(B+C)\right]}=\frac{\tan\left[\frac{1}{2}(b-c)\right]}{\tan\left[\frac{1}{2}(b+c)\right]}$

Right Angled Spherical Triangle

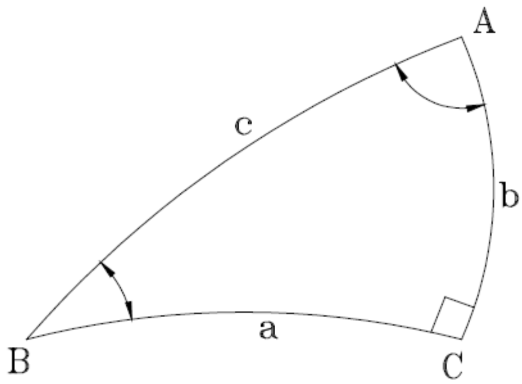

Fig.3 (a) Right Spherical Triangle

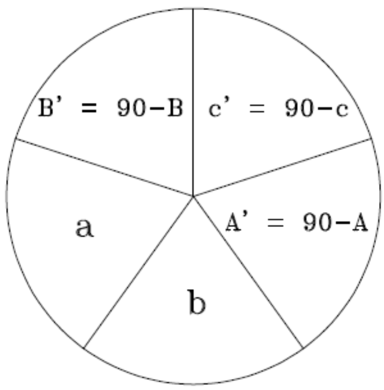

Fig.3 (b) Napier's Circular Parts

Napier's Circle (see Fig.3 (b)) shows the relations of parts of a right spherical triangle (See Fig. 3(a)), which helps to find all relations between the angles in a right spherical triangle. In Fig. 3(a), Let ABC is a right spherical triangle whose right angle is at C. To construct the Napier's Circle mark the six angles of the triangle (three corner angles A, B, C, three arc angles a, b, c) in the form of a circle, sticking to the order as they appear in the triangle (i.e., start with a corner angle, write the arc angle of an adjacent side next to it, proceed with the next corner angle, etc. and close the circle). Then eliminate the 90° corner angle and replace all angles non-adjacent to it by their complement to 90° (i.e. replace, say, A' by 90° – A). Now we have the five parts of the right spherical triangle representing in the Napier's Circle shown in Fig. 3(b).

For any choice of the angle (called middle angle) will have two side angles on either side next to it (called adjacent angles) and other two remaining angles called opposite angles. Then as per the Napier's Rules that the Sine of the middle angle is equal to,

The product of the Tangents of the adjacent angles --- Rule-1

The product of the Cosines of the opposite angles --- Rule-2

The application of these rules can be seen in the derivation of necessary trigonometric equations for the house cusps shown in this article.

Solution for the oblique spherical triangle

In the case of oblique spherical triangle, when any there parts are known then the remaining parts may be found either by dividing the oblique spherical triangle into two right spherical triangle for using Napier's circular part Rules or using the four formulas may be referred to as the sine Rule, the cosine Rule for side, the cosine rule for Angle, and the cotangent Rule. Also with 5 parts cosine rules. All these rules are already given under the heading "Napier's Rules for Spherical Triangle".

The art of solving a spherical triangle entirely depends on understanding which rule is appropriate under given circumstances. Each rule contains four elements (angles and sides) and any three of them may be assumed to be known, and the fourth is to be determined. For the reader's easy reference and use, a table is give below showing the rule that is associated with specific spherical triangle problems.

Rule	Problem involving in a spherical triangle
Sine Rule	Any two corner angles and it's opposite side
Cosine Rule (Side)	All three sides with any one corner angle
Cosine Rule (Angle)	All three corner angles with any one side
Cotangent Rule	Any four consecutive parts (Side-Angle-Side-Angle)
Cosine Rule (Except 1 Angle)	All three sides with any two corner angle
Cosine Rule (Except 1 Side)	All three corner angles with any two side

The sine and the cotangent rules are particularly useful and frequently needed for our computation. But unfortunately the cotangent rules are very difficult to remember it and also difficult to decide from the rule for choosing the sides and angles. However it may be noted that the cotangent rule shows the connection between four consecutive parts of the spherical triangle as shown in the Figure

4(a) & (b). Now the angle subtended between the two consecutive sides may be called as inner angle (IA), the side subtended between the two consecutive angles may be called as inner side (IS) and the rest of the angle and side may be called as outer angle (OA) and outer side (OS) respectively. We can easily write the cotangent rule as given below.

$$\cos(IS)\cos(IA) = \cot(OS)\ \sin(IS) - \cot(OA)\sin(IA)$$

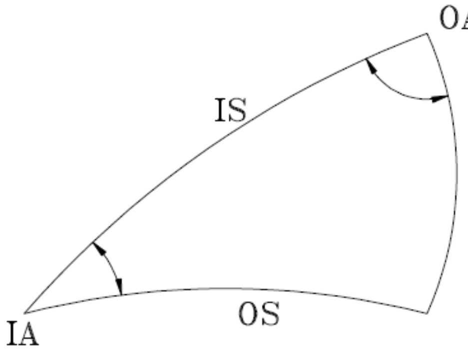

Fig.4 (a) Spherical Triangle

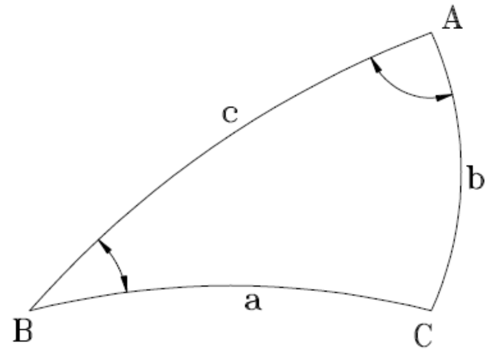

Fig.4 (b) Spherical Triangle Parts

For example, in writing down the rule involving the four parts a, c, A, B we have B as the inner angle (IA) and c as the inner side (IS), A as the outer angle (OA) and **a** as the outer side (OS), we obtain cotangent rule as,

$$\cos c \cos B = \cot a \sin c - \cot A \sin B$$

An introdution to Astronomy

Before we go into the formulation of the house cusps we need to understand about the astronomy and some of the terms used in it. Though more details may be found in many astronomy/physics (Theory of Relativity and Quantum Mechanics) books and other online resources, for the reader's ready reference a brief information is given here.

Astronomy is a science that deals with the study of celestial objects (such as stars, planets, comets etc) and their behavior. It is more used in physics, space navigation, and motion of the celestial objects etc.

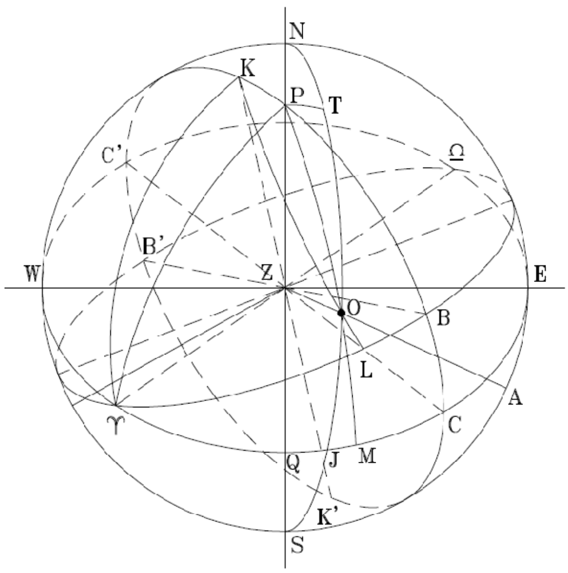

Fig.5 (a) Celestial Sphere (Top View)

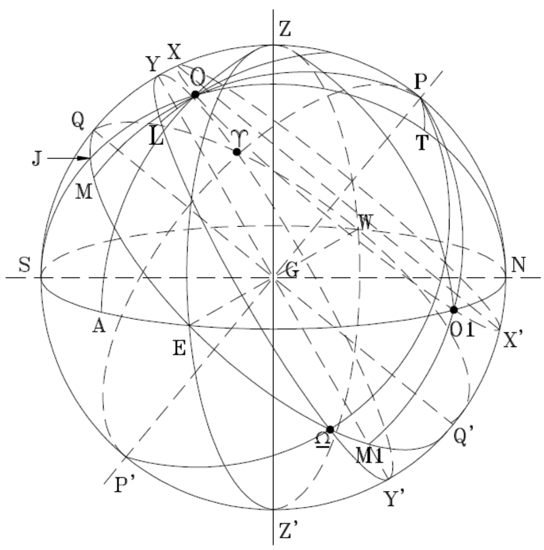

Fig.5 (b) Celestial Sphere (East View)

As the reader may note that the part of ecliptic latitude circle LO in Fig. 5(b) does not have ecliptic pole K on the meridian why? Because it is not laying in the meridian plane. The Pole K will appear in the meridian only when the solstitial colure (PCK'C') coincides with the meridian. Some books/literature/online resource may show that all the three points Z, P & K are lies in the meridian and the readers must understand carefully whether it is an indicative or actual situation as explained above.

Description of the Notations

Item (Part)	Description	Remarks
AO	Altitude of the body at O	Fig.5 (a) & 5 (b)
EA, EZA	Amplitude of the body at O	Fig.5 (a) & 5 (b)
JM, EM_1	Ascensional difference of a body at O & O_1	Fig.5 (a) & 5 (b)
SA, SZA	Azimuth of the body at O	Fig.5 (a) & 5 (b)
NOS	Cardinal Circle of the body at O	Fig.5 (a) & 5 (b)
PT	Cardinal Pole of the body at O	Fig.5 (a) & 5 (b)
♈Q♎Q'	Celestial Equator	Fig.5 (b)
NWSE	Celestial Horizon	Fig.5 (a) & 5 (b)
ZP	Co-Latitude of the place	Fig.5 (a) & 5 (b)
MO	Declination of the body at O	Fig.5 (a) & 5 (b)
M_1O_1	Declination of the body at O_1	Fig.5 (b)
OXX'O_1	Diurnal Circle of the body O	Fig.5 (b)
♈L	Ecliptic longitude	Fig.5 (a) & 5 (b)
♈B♎B'	Ecliptic	Fig.5 (a)
♈Y♎Y'	Ecliptic	Fig.5 (b)
LO	Ecliptic latitude	Fig.5 (a) & 5 (b)
KOL	Ecliptic latitude Circle (Part of the latitude Circle)	Fig.5 (a)
P♈P'♎	Equinoctial colure	Fig.5 (b)
QM, QPM	Hour Angle of the body at O	Fig.5 (a) & 5 (b)
Q M_1,QPM_1	Hour Angle of the body at O_1	Fig.5 (b)
PN	Latitude of the place	Fig.5 (a) & 5 (b)
Q'M	Meridian Distance (Lower) of the body at O	Fig.5 (b)
Q'M_1	Meridian Distance (Lower) of the body at O_1	Fig.5 (b)
Q M	Meridian Distance (Upper) of the body at O	Fig.5 (a) & 5 (b)
Q M_1	Meridian Distance (Upper) of the body at O_1	Fig.5 (b)
NZSZ'	Meridian of the place	Fig.5 (b)
P, P'	North & South Celestial Poles	Fig.5 (b)
K, K'	North & South Ecliptic Poles	Fig.5 (a)
N, S, E, W	North, South East & West point in the Horizon	Fig.5 (a) & 5 (b)
♈J	Oblique Ascension of the body at O	Fig.5 (a) & 5 (b)
PK,B♈C	Obliquity of the ecliptic	Fig.5 (a)
Y♈Q	Obliquity of the ecliptic	Fig.5 (b)
PO	Polar Distance of the body at O	Fig.5 (a) & 5 (b)
PO_1	Polar Distance of the body at O_1	Fig.5 (b)
PT	Pole Height of the body at O (Arc Distance)	Fig.5 (a) & 5 (b)
WZEZ'	Prime Vertical	Fig.5 (b)
♈M, ♈M_1	Right Ascension of the body at O & O_1	Fig.5 (a) & 5 (b)
XO_1, QM_1	Semi-Diurnal Arc of a body at O & O_1	Fig.5 (b)
X'O_1, Q'M_1	Semi-Nocturnal Arc of a body at O & O_1	Fig.5 (b)

Item (Part)	Description	Remarks
♈Q, ♈PQ	Sidereal Time of the body	Fig.5 (a) & 5 (b)
B, B'	Solstices of the ecliptic	Fig.5 (a)
PCK'C'	Solstitial colure	Fig.5 (a)
♈, ♎	Vernal equinox, autumnal equinox	Fig.5 (a) & 5 (b)
Z, Z'	Zenith & Nadir	Fig.5 (b)
ZO	Zenith Distance of the body at O	Fig.5 (a) & 5 (b)

Two different views of the Celestial Sphere are shown in the above Figure 5 for better understanding. The one looking below from the top of the observer's Head is shown in Figure 5(a) (Top view) and the other view is standing on the horizon looking from East side is shown in Figure 5(b) (East View). The readers may refer any good textbooks on astronomy or even on-line resources showing the 3D color views even with animation for more clarity & better understanding.

Celestial sphere

It is an imaginary sphere on whose surface all the stars and other celestial bodies are appear to be embedded is called the celestial sphere (See Fig 5(a) & 5(b)). In order to determine the position of a celestial body on the celestial sphere and express the relation existing between two or more celestial bodies, the circles, points, lines and terms defined further below.

Great circle

Any plane passing through the center of the celestial sphere (G in Fig 5(b)) bisects the sphere into two equal hemispheres is called great circle.

Celestial Horizon, zenith, Nadir, Vertical Circles

The **celestial horizon** (horizon- NWSE in Fig. 5(a)) is the great circle of the celestial sphere whose plane passes through the center of the celestial sphere (G in Fig 5(b)) and parallel to the observer's horizontal plane (called terrestrial horizon). A plumb line that passes through the observer's head and toe will penetrate the celestial sphere at a point above the horizon is called **zenith** (Point Z in Fig. 5(a) & 5(b)) and the point below is called **nadir** (Point Z' in Fig. 5(b)). The zenith and nadir are the poles of the horizon, and all great circles passing through them are called **vertical circles**.

Points of the horizon, Meridian & Prime vertical

The **points of the horizon** directly east, west, north and south of the observer are called, respectively, the east, west, north and south points (Point E, W, N, S in Fig. 5(a) & 5(b)). The **meridian** (NZSZ' in Fig. 5(b)) is the vertical circle which passes through the south and north points of the horizon. The **prime vertical** (WZEZ' in Fig. 5(b)) is the vertical circle which passes through the east and west points of the horizon.

Altitude, Zenith distance, Azimuth & Amplitude

The **altitude** (AO in Fig. 5(a) & 5(b)) of a celestial body is its distance from the horizon, measured on the vertical circle passing through the celestial body. Distances above the horizon are measured +ve and below are –ve. The altitudes of all points on the celestial sphere will always lies between +90 to –90 only. It may be denoted by $\mathbf{A_L}$.

The distance of the celestial body from the zenith, measured on the vertical circle of the celestial body is called the **zenith distance** (ZO in Fig. 5(a) & 5(b)). It is the complement of the altitude. The zenith distances of all celestial bodies on the sphere lie between 0 and + 180. It is denoted by **z.**

The **azimuth** (SA, SZA in Fig. 5(a) & 5(b)) of a celestial body is the arc of the horizon intercepted between the vertical circle of the celestial body and the nearest north or south points on the horizon. Generally the azimuth is measured from the north point around to the east through 360. It is denoted by **A.**

The **amplitude** (EA, EZA in Fig. 5(a) & 5(b)) of a celestial body is the arc of the horizon intercepted between the vertical circle of the celestial body and the nearest east or west points on the horizon. It is the complement of the azimuth.

The Azimuth and altitude are usually used together to give the direction of the celestial object in the Horizontal (topocentric) coordinate system.

Celestial equator, Poles of the equator & Hour Circle/Declination Circle

The **celestial equator** (♈Q♎Q' in Fig. 5(b)) is the great circle of the sphere whose plane is parallel to the terrestrial equator and perpendicular to the earth's axis which cuts the celestial sphere into two equal hemisphere the upper part is called northern hemisphere and the lower part is called southern hemisphere.

The earth's axis produced is the axis of the celestial sphere penetrates the celestial sphere in the northern hemisphere and southern hemisphere called the **North Pole** (Point P in Fig. 5(b)) and **South Pole** (Point P' in Fig. 5(b)) respectively.
All great circles passing through the north and south poles are called **hour circles** or **declination circles**. The hour circle passing through either zenith or nadir coincides with the meridian.

Declination, Polar distance & Hour Angle

The **declination** (MO, M_1O_1 in Fig. 5(a) & 5(b)) of a celestial body is its distance from the celestial equator, measured on the hour circle passing through the celestial body. Distances measured north are +ve and south is -ve. The declination of all points on the celestial sphere will always lies between –90 to +90 only. It is denoted by **δ.**

The distance of the celestial body from the North Pole, measured on the hour circle of the celestial body is called the **polar distance** (PO, PO_1 in Fig. 5(a) & 5(b)). It is the complement of the declination. The polar distances of the celestial bodies on the celestial sphere lies between 0 to 180.

The **hour angle** (QM, QM_1 in Fig. 5(a) & 5(b)) of a celestial body is the arc of the celestial equator intercepted between the meridian, or south point of the celestial equator, and the hour circle passing through the celestial body. Hour angle is considered as the equivalent angle at the North Pole (QPM, QPM_1 in Fig. 5(a) & 5(b)) between the meridian and hour circle. It is considered from the meridian around to the west through 24 hours, or 360. It is denoted by **H.**

The Hour angle and declination are usually used together to give the direction of the celestial object in the equatorial coordinate system.

Ecliptic, Equinoxes, Solstices, Solstitial colure & obliquity

The **ecliptic** (♈B♎B' in Fig. 5(a)) is the great circle of the celestial sphere formed by the plane of the earth's orbit or, it is the great circle described by the apparent annual motion of the sun. It intersects the celestial equator in two points called the **equinoxes** (♈, ♎ in Fig. 5(a) & 5(b)).

The **vernal equinox** (also called First point of Aries ♈ in Fig. 5(a) & 5(b)) is that point through which the sun appears to pass in going from the south to the north side of the celestial equator. The **autumnal equinox** (also called First point of Libra ♎ in Fig. 5(a) & 5(b)) is that point through which the sun appears to pass in going from the north to the south side of the equator.

The **solstices** (B, B' in Fig. 5(a)) are the points of the ecliptic 90 degree from the equinoxes and at these points the declination of the sun will be maximum. The **equinoctial colure** (P♈P'♎ in Fig.5 (b)) is the hour circle passing through the equinoxes. The **solstitial colure** (PCK'C' in Fig.5 (a)) is the hour circle passing through the solstices.

The angle between the celestial equator and ecliptic is called the **obliquity** (PK, B♈C, Y♈Q in Fig. 5(a) & 5(b)) of the ecliptic. This slightly various with respect to each year and may be calculated from the mathematical models/equations that are formulated based on the observation records. It is denoted by **ε.**

Right ascension, Sidereal Time, Latitude Circles, Ecliptic Latitude/Longitude

The **right ascension** (♈M, ♈M_1 in Fig. 5(a) & 5(b)) of a celestial body is the arc distance of the celestial equator intercepted between the vernal equinox and the hour circle of the celestial body. It is measured from the vernal equinox toward the east through 24 hours, or 360. It is denoted by **α.**

The **sidereal time** (♈Q, ♈PQ in Fig. 5(a) & 5(b)) at any point of observation is equal to the right ascension of the observer's meridian. It is equal to the hour angle of the vernal equinox. Time based on the rotation of the Earth with respect to the background of fixed stars. Astronomers generally use sidereal time instead of solar time for the observation of the celestial bodies. It is denoted by **T.**

Great circles perpendicular to the ecliptic are called **latitude circles** (KOL in Fig. 5(a) is the part circle). The **ecliptic latitude** (LO in Fig. 5(a) & 5(b)) of a celestial body is its distance from the ecliptic, measured on the latitude circle passing through the celestial body. Distances measured north are +Ve and south are -Ve. The latitudes of all points on the celestial sphere are always between –90 to +90 only. When we consider the Sun as the celestial body then it will be always zero as it's apparent movement is always in the same ecliptic plane. It is denoted by **β.**

The **ecliptic longitude** (♈L in Fig. 5(a) & 5(b)) of a celestial body is the arc of the ecliptic intercepted between the vernal equinox and the latitude circle of the celestial body. It is measured from the vernal equinox toward the east through 360. It is denoted by **λ.**

The ecliptic latitude and ecliptic longitude are used together to give the direction of the celestial object in the ecliptic coordinate system. Similarly the right ascension and declination are used to give the direction of the celestial object in the equatorial coordinate system.

Cardinal Circle, Oblique Ascension, Oblique Descension, Pole Height/Polar Elevation

The **Cardinal Circle** (NOS in Fig. 5(a) & 5(b)) of the celestial body is the great circle passing through the celestial body from the north and south points of the celestial horizon.

When the celestial body is to the east of the meridian of the place then the **Oblique Ascension** (♈J in Fig. 5(a) & 5(b)) of the celestial body is the arc distance measured along the celestial equator between the vernal equinox point and the point at which the cardinal circle of the celestial body intersects the celestial equator. It may be denoted by **OA** or **a.**

When the celestial body is to the west of the meridian of the place then the **Oblique Descension** of the celestial body is the arc distance measured along the celestial equator between the vernal equinox point and the point at which the cardinal circle of the celestial body intersects the celestial equator. It may be denoted by **OD.**

The **Pole** of the celestial body is the arc of a great circle drawn perpendicular to the cardinal circle of the celestial body and joining the nearer celestial pole. The readers should note that we have already discussed about the poles earlier particularly celestial poles which is totally different from the pole mentioned here. To avoid confusion the pole described here may be called as **Cardinal Pole** (PT in Fig. 5(a) & 5(b)). The arc length of the Pole (Cardinal Pole) is called **Pole Height** or **Polar Elevation** (PT in Fig. 5(a) & 5(b)) of the celestial body. It may be denoted by **p.**

When the celestial body is at the celestial horizon then the pole height of the celestial body will be equal to the latitude of the observer's place (which is equal to the altitude of the observer's place) because the cardinal circle of the celestial body will coincide with celestial horizon at the same time the perpendicular arc line drawing from the nearer celestial pole will coincide with the meridian. Similarly when the celestial body is at the meridian then the pole height of the celestial body will be equal to zero because the cardinal circle of the celestial body will coincide with the meridian. More details may be found in the following sections of this article.

Meridian Distance, Ascensional difference

The **Meridian Distance** (may be denoted by $\mathbf{M_d}$) of a celestial body is the arc distance measured along the celestial equator between the upper or lower meridian of the place and the point at which the declination circle of the celestial body intersects the celestial equator. When the distance is measured from upper meridian then it is called **upper meridian distance** (QM, QM_1 in Fig. 5(a) & 5(b)) and similarly, when the distance is measured from the lower meridian of the place then it is called **lower meridian distance** (Q'M, $Q'M_1$ in Fig. 5(b)).

The **Ascensional difference** (JM, EM_1 in Fig. 5(a) & 5(b)) of a celestial body is the arc distance measured along the celestial equator between the cardinal circle and the declination circle of the celestial body. It is nothing but the difference in celestial equatorial distance between the right ascension and the oblique ascension of the celestial body. When the celestial body is at the meridian then the Ascensional difference of the celestial body will be equal to zero because both the cardinal circle and the declination circle of the celestial body will coincide with the meridian. Similarly when the celestial body is at the celestial horizon then the Ascensional difference of the celestial body will be maximum. More details may be found in the following sections of this article. It may be denoted by **h** or **AD**

Semi-Arcs, Diurnal Motion, Diurnal Circle & Circumpolar Star

The **SemiArc** of a celestial body is the arc distance measured along the celestial equator between the upper or lower meridian of the place and the point at which the declination circle of the celestial body at horizon intersects the celestial equator. When the distance is measured from upper meridian then it is called **Semi-Diurnal Arc** (QM_1 in Fig. 5(b)) and similarly, when the distance is measured from the lower meridian of the place then it is called **Semi-Nocturnal Arc** ($Q'M_1$ in Fig. 5(b)). In fact it is the Hour Angle of the celestial body when it is on the horizon of a place. Also it is nothing but the Meridian Distance of a celestial body when it is on the horizon of a place. The Semi-Diurnal Arc and the Semi-Nocturnal Arc may be denoted by **H** & **H'** respectively.

Diurnal motion is the apparent daily motion of celestial bodies around the Earth, or more particularly around the two celestial poles. It is caused by the Earth's rotation on its axis, so every celestial body apparently moves on a circle that is called the **diurnal circle** ($OXX'O_1$ in Fig. 5(b)). It is nothing but the path traced by the celestial body on the celestial sphere due to the earth's rotation. Also all the celestial bodies will rise and set in the horizon on every day except when they are circumpolar.

A **circumpolar star** is a star that, as viewed from a given latitude of the place on Earth, will never either disappears below the horizon (i.e., never sets) or appears above the horizon (i.e., never rises), due to its proximity to one of the celestial poles.

Geographical Latitude & Geographical Longitude

The position of an observer on the earth's surface is defined by his geographical latitude and longitude. The **geographical latitude** (It is denoted by ϕ) of a place is the declination of the zenith of the place (NP in Fig. 5(b)). It is also equal to the altitude of the North Pole. Latitudes of places north of the earth's equator are +ve and south are -ve. The arc distance of the meridian from the Pole to Zenith of the place is called **Co-latitude** of a place (ZP in Fig. 5(b)). The **geographical longitude** of a place is the arc of the earth's equator intercepted between the meridian of the place (The Vertical circle passing through the North, South and the observer's place on the earth) and the meridian of some other reference place (Greenwich) assumed as origin. It is a normal practice to consider the western longitudes are (-ve) and eastern longitudes are (+ve) from the meridian of Greenwich, through 12 hours, or 180. Geographical latitude and longitude are generally used for calculating the cuspal positions, rise/set of the celestial bodies etc.

The relation between ecliptic (λ,β) and equatorial (α,δ) coordinates

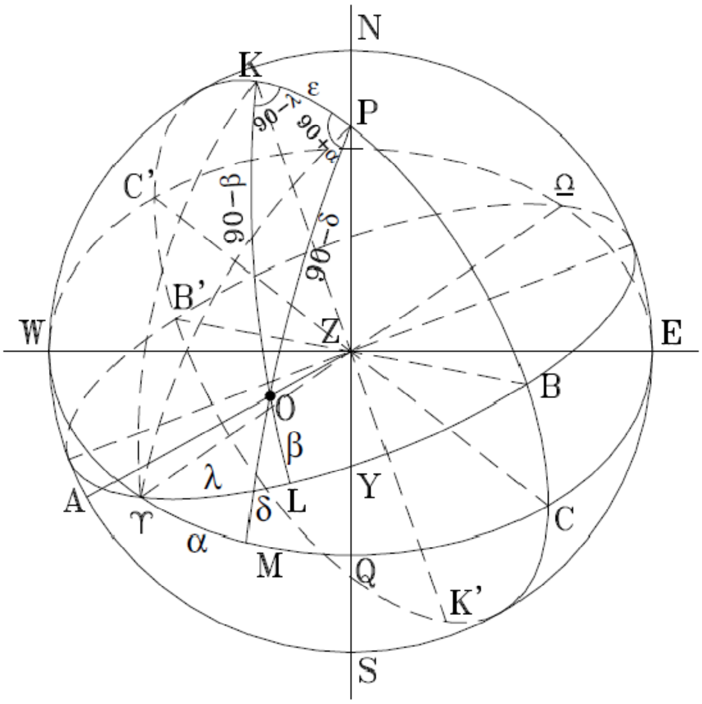

Fig.6) Celestial Sphere (Top View) & Spherical Triangle PKO

In Fig. 6, all the notations used are same as Fig. 5(a) except the position of the celestial body O. In Spherical Triangle PKO Let,

$\Upsilon M = \Upsilon PM = \boldsymbol{\alpha}$ (Right Ascension of the celestial body O)
$\Upsilon L = \Upsilon KL = \boldsymbol{\lambda}$ (Ecliptic longitude of the celestial body O)
$OM = \boldsymbol{\delta}$ (Declination of the celestial body O)
$OL = \boldsymbol{\beta}$ (Ecliptic latitude of the celestial body O)
$KP = \boldsymbol{\varepsilon}$ (obliquity of the ecliptic)
$\Upsilon KL + LKB = 90$
$LKB = 90 - \Upsilon KL$
$LKB = 90 - \boldsymbol{\lambda}$
$KPM = KP\Upsilon + \Upsilon PM$
$KPM = 90 + \boldsymbol{\alpha}$
$PO + OM = 90$

PO = 90 - δ

KO + OL = 90

KO = 90 - β

From the Spherical Triangle PKO,

Apply the cosine rule for the side OP:

$$\cos(90°-\delta) = \cos(90°-\beta)\cos(\varepsilon) + \sin(90°-\beta)\sin(\varepsilon)\cos(90°-\lambda)$$

$$\sin(\delta) = \sin(\beta)\cos(\varepsilon) + \cos(\beta)\sin(\varepsilon)\sin(\lambda) \qquad \text{---Eqn.(1)}$$

Apply the cosine rule for 5 Parts Except one Angle KOP:

$$\sin(90°-\beta)\cos(90°-\lambda) = \cos(90°-\delta)\sin(\varepsilon) - \sin(90°-\delta)\cos(\varepsilon)\cos(90°+\alpha)$$

$$\cos(\beta)\sin(\lambda) = \sin(\delta)\sin(\varepsilon) + \cos(\delta)\cos(\varepsilon)\sin(\alpha) \qquad \text{---Eqn.(2)}$$

Apply the cosine rule for the side OK:

$$\cos(90°-\beta) = \cos(90°-\delta)\cos(\varepsilon) + \sin(90-\delta)\sin(\varepsilon)\cos(90°+\alpha)$$

$$\sin(\beta) = \sin(\delta)\cos(\varepsilon) - \cos(\delta)\sin(\varepsilon)\sin(\alpha) \qquad \text{---Eqn.(3)}$$

Apply the Sine rule for the two corner angles and it's side,

$$\sin(90°-\beta)\sin(90°-\lambda) = \sin(90°+\alpha)\sin(90°-\delta)$$

$$\cos(\beta)\cos(\lambda) = \cos(\alpha)\cos(\delta) \qquad \text{---Eqn.(4)}$$

Dividing Eqn.(2) by Eqn.(4) we get,

$$\tan(\lambda) = \frac{\sin(\delta)\sin(\varepsilon) + \cos(\varepsilon)\cos(\delta)\sin(\alpha)}{\cos(\alpha)\cos(\delta)} \qquad \text{---Eqn.(5)}$$

Eqn.5 may be further simplified in to,

$$\tan(\lambda) = \frac{\tan(\delta)\sin(\varepsilon) + \cos(\varepsilon)\sin(\alpha)}{\cos(\alpha)} \qquad \text{---Eqn.(6)}$$

The above Eqn.5 may be written in another form by using the following auxiliary components. Let,

$$\mathrm{N}\sin \mathrm{M} = \sin(\delta) \qquad \text{---Eqn.(7)}$$

$$\mathrm{N}\cos \mathrm{M} = \cos(\delta)\sin(\alpha) \qquad \text{---Eqn.(8)}$$

Using values from Equation 7, 8 in Eqn. 5, we get,

$$\tan(\lambda) = \frac{\mathrm{N}\sin(M)\sin(\varepsilon) + \mathrm{N}\cos(M)\cos(\varepsilon)}{\cos(\alpha)\cos(\delta)} \qquad \text{---Eqn.(9)}$$

$$\tan(\lambda) = \frac{\mathrm{N}\cos(M-\varepsilon)}{\cos(\alpha)\cos(\delta)} \qquad \text{---Eqn.(10)}$$

From Eqn.8, We get,

$$\cos(\delta) = \frac{\mathrm{N}\cos(M)}{\sin(\alpha)} \qquad \text{---Eqn.(11)}$$

Using values from Equation 11 in Eqn. 10, we get,

$$\tan(\lambda) = \frac{\mathrm{N}\cos(M-\varepsilon)\sin(\alpha)}{\mathrm{N}\cos(\mathrm{M})\cos(\alpha)} \qquad \text{---Eqn.(12)}$$

$$\tan(\lambda) = \frac{\cos(M-\varepsilon)\tan(\alpha)}{\cos(M)} \qquad \text{---Eqn.(13)}$$

Dividing Eqn.(7) by Eqn.(8) we get,

$$\tan(M) = \frac{\tan(\delta)}{\sin(\alpha)} \qquad \text{---Eqn.(14)}$$

In astrology, we use sun as the celestial body for computing the house cusp longitudes. Since the sun is always moving in the ecliptic plane, the ecliptic latitude (β) will be zero. So once we know the equatorial coordinates (α,δ) i.e., right ascension & the declination of the celestial body, we can compute the ecliptic longitude (λ) for the celestial body by using either eqn.6 or the set of Eqns. 13 & 14.

Many years ago may be a century ago, the values are computed by using the logarithmic tables for which the equations 13 & 14 are found more convenient. But nowadays we have scientific calculators/computers, which will do the job very fast.

As the sun in always moving in the ecliptic plane, the ecliptic latitude (β) is zero. So the object O shown in Fig.6 will be on the ecliptic plane as shown in Fig. 7(a) below. Thus right angled spherical triangle ♈MO can be solved by using the Napier's Circular Parts (shown in Fig. 7(b)) Rules.

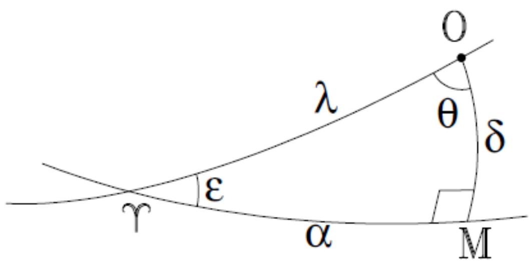

Fig.7 (a) Right Spherical Triangle

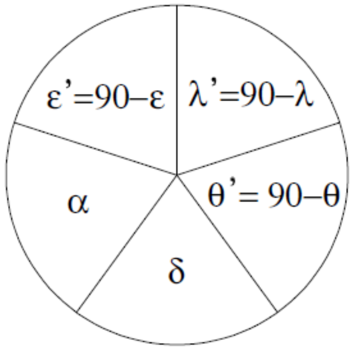

Fig.7 (b) Napier's Circular Parts

We have two rules and five parts for which a set of equations may be derived as shown below. Considering the length of the document the step by step formulation is not given and only the summaries of equation are shown below. The readers may derive themselves by applying the Napier's Circular parts Rule1 & 2. However for typical application of these rules, the readers may refer the derivation of trigonometric equations for the house cusps shown in this article.

Summary of Right Ascension Equations

Sr.No.	Equations	Remarks
1	$\sin(\alpha) = \dfrac{\tan(\delta)}{\tan(\varepsilon)}$	---Eqn.(15)
2	$\sin(\alpha) = \sin(\lambda)\sin(\theta)$	---Eqn.(16)
3	$\cos(\alpha) = \dfrac{\cos(\theta)}{\sin(\varepsilon)}$	---Eqn.(17)
4	$\cos(\alpha) = \dfrac{\cos(\lambda)}{\cos(\delta)}$	---Eqn.(18)
5	$\tan(\alpha) = \sin(\delta)\tan(\theta)$	---Eqn.(19)
6	$\tan(\alpha) = \tan(\lambda)\cos(\varepsilon)$	---Eqn.(20)

Summary of Declination Equations

Sr.No.	Equations	Remarks
1	$\sin(\delta)=\dfrac{\tan(\alpha)}{\tan(\theta)}$	--Eqn.(21)
2	$\sin(\delta)=\sin(\lambda)\sin(\varepsilon)$	--Eqn.(22)
3	$\cos(\delta)=\dfrac{\cos(\lambda)}{\cos(\alpha)}$	--Eqn.(23)
4	$\cos(\delta)=\dfrac{\cos(\varepsilon)}{\sin(\theta)}$	--Eqn.(24)
5	$\tan(\delta)=\sin(\alpha)\tan(\varepsilon)$	--Eqn.(25)
6	$\tan(\delta)=\tan(\lambda)\cos(\theta)$	--Eqn.(26)

Summary of Ecliptic longitude Equations

Sr.No.	Equations	Remarks
1	$\sin(\lambda)=\dfrac{\sin(\alpha)}{\sin(\theta)}$	--Eqn.(27)
2	$\sin(\lambda)=\dfrac{\sin(\delta)}{\sin(\varepsilon)}$	--Eqn.(28)
3	$\cos(\lambda)=\dfrac{1}{\tan(\theta)\tan(\varepsilon)}$	--Eqn.(29)
4	$\cos(\lambda)=\cos(\delta)\cos(\alpha)$	--Eqn.(30)
5	$\tan(\lambda)=\dfrac{\tan(\delta)}{\cos(\theta)}$	--Eqn.(31)
6	$\tan(\lambda)=\dfrac{\tan(\alpha)}{\cos(\varepsilon)}$	--Eqn.(32)

Summary of Corner Angle Equations

Sr.No.	Equations	Remarks
1	$\sin(\theta)=\dfrac{\sin(\alpha)}{\sin(\lambda)}$	--Eqn.(33)
2	$\sin(\theta)=\dfrac{\cos(\varepsilon)}{\cos(\delta)}$	--Eqn.(34)
3	$\cos(\theta)=\dfrac{\tan(\delta)}{\tan(\lambda)}$	--Eqn.(35)
4	$\cos(\theta)=\sin(\varepsilon)\cos(\alpha)$	--Eqn.(36)
5	$\tan(\theta)=\dfrac{\tan(\alpha)}{\sin(\delta)}$	--Eqn.(37)
6	$\tan(\theta)=\dfrac{1}{\cos(\lambda)\tan(\varepsilon)}$	--Eqn.(38)

Note:

The art of solving a right angled spherical triangle entirely depends on understanding which rule is appropriate under given circumstances. Each rule contains three elements (angles and sides) and any two of them may be assumed to be known, and the third is to be determined.

Definition of Zodiac & The House Cusps

We have seen the details/definition of the basic terms used in astronomy and also the formulas/rules used in the trigonometry (both plane & Spherical). Before we go into the house cusp formulation let us understand the definition of the zodiac and the house cusps used in the astrology.

We all aware that the **zodiac** is nothing but the ecliptic plane divided in the 12 equal parts each having 30 degree interval called Signs. The first 6 (six) signs are considered starting from First point of Aries to First point of Libra covering 180 degree and the balance 6 (six) signs are considered for the remaining part(180 degree) of the ecliptic.

The zodiac points are independent of the place and time for the western system (Sayana system) in which a movable zodiac in considered. But in the Hindu/Nirayana system the zodiac is considered to be fixed where the time factor affects the position of the First point of Aries with respect to fixed star position called precession (or Ayanamsa). Hence for the particular date/time this precession (or Ayanamsa) must be computed in order to find the starting/original zodiac position considered for Zero degree precession (or Ayanamsa) date. So for the Hindu/Nirayana system the sayana (movable) longitude of Cusp & planets must be deducted with the precession (or Ayanamsa) to get the longitudes corresponding to fixed zodiac. For more details about precession (or Ayanamsa) separate literate may be referred.

The **house cusps** are the 12(twelve) points on the ecliptic whose longitude on the ecliptic needs to be known for the particular birth place. These house cusp point's longitude various with respect to time and place. There are many house division methods such as Alchabitius, Campanus, Koch, Meridian, Placidus, Porphyry, Regiomontanus, Topocentric etc are available to get the longitude of the house cusps in the ecliptic and here our interest is about the placidus house the most commonly used house system in modern Western astrology & KP (Krishnamurti Paddhati) System of Astrology.

Placidus house system is an unequal house system. Depending on the latitude considered, a Placidus chart may have entire signs contained in a house, known as intercepted signs. Charts drawn for higher latitudes often have house intervals that are either very small or very large and may be more or less equal (less difference) when the latitude considered is closer to the equator. This is because the diurnal circle of a body will have more duration of time to transit from the raising point (on the eastern side of the horizon) to setting point (on the western side of the horizon) and less duration of time form setting point (on the western side of the horizon) to the raising point (on the eastern side of the horizon). Thus the hour angle or the Semi-Diurnal Arc and Semi-Nocturnal Arc length of a body will be extremely unequal. When these two extremely unequal parts are trisected then obviously the house cusp intervals computed will have higher difference. This difference in Semi-Arc length will reduce gradually when we go closer to the equator and certainly the house cusp intervals computed will have lesser difference. Also depending on the declination of the celestial body (Sun) the Semi-Arc lengths will get affected due to high (Maximum positive/negative values) declination and low declination zero. This may be visualized from Fig.8 shown below.

The Placidus system is not possible to apply beyond latitudes greater than 66.5°N or 66.5°S, because the celestial body (sun) will be in circumpolar state and there is no diurnal circle with rise & set points for the celestial body (sun) can be found to trisect. This is a drawback in the Placidus system.

Placidus House Cusp (Semi-Arc Method)

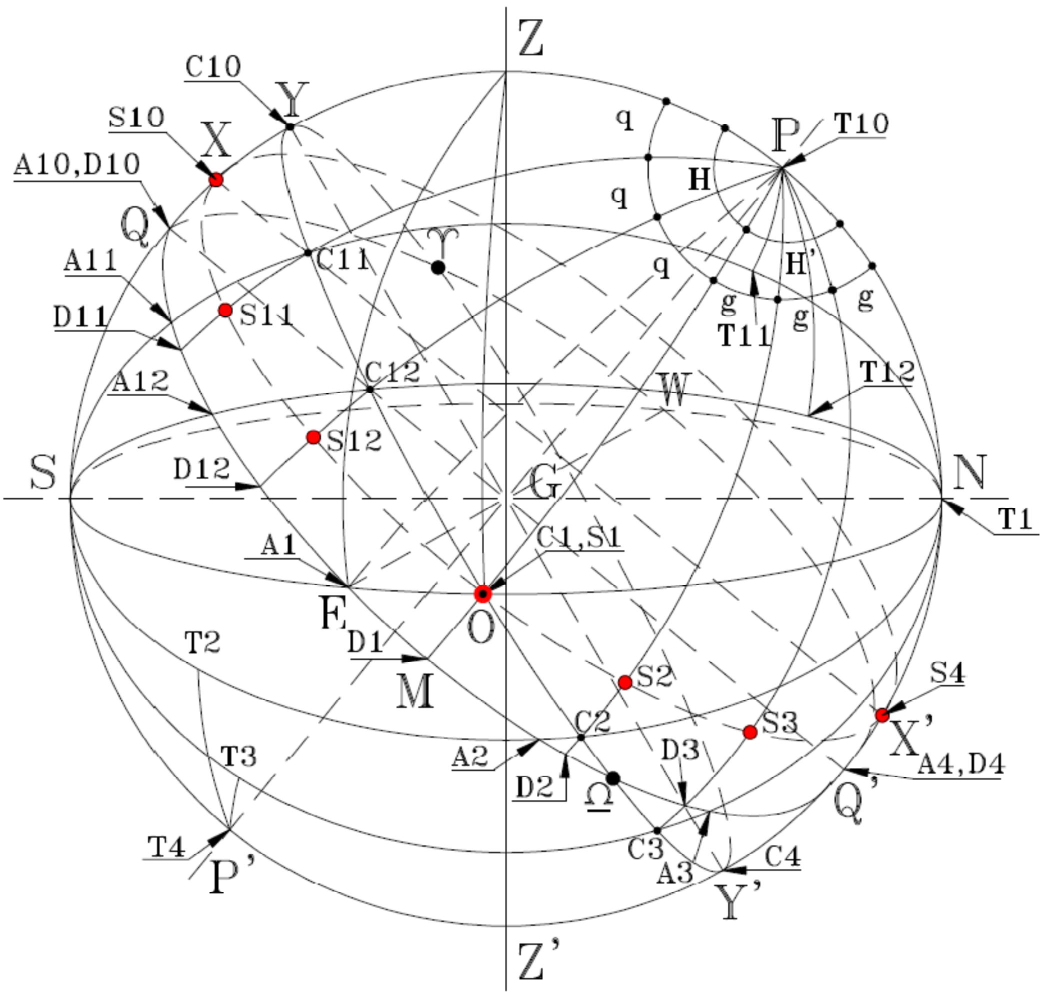

Fig.8) Celestial Sphere & the House Cusps

Description of the Notations:

Item (Part)	Description	Remarks
EO, EZO	Amplitude of the Sun when it is at (S_1)ascendant	
S_{10}, S_{11}, S_{12}, S_1, S_2, S_3	Apparent position of the Sun in the diurnal circle corresponding to the Cusp points C_{10}, C_{11}, C_{12}, C_1, C_2 & C_3 respectively. Where S_1 is the Sun at Ascendant.	
$A_{10}D_{10}$, $A_{11}D_{11}$, $A_{12}D_{12}$, A_1D_1, A_2D_2, A_3D_3	Ascensional difference for the Cusp points C_{10}, C_{11}, C_{12}, C_1, C_2 & C_3 respectively	
EM	Ascensional difference for sun when it is at (S_1) ascendant	
NO, NZO	Azimuth of the Sun when it is at (S_1)ascendant	
$NC_{10}S$, $NC_{11}S$, $NC_{12}S$, NC_1S, NC_2S, NC_3S	Cardinal Circle for the Cusp points C_{10}, C_{11}, C_{12}, C_1, C_2 & C_3 respectively	
PT_{10}, PT_{11}, PT_{12}, PT_1, $P'T_2$, $P'T_3$	Cardinal Pole for the Cusp points C_{10}, C_{11}, C_{12}, C_1, C_2 & C_3 respectively	
♈Q♎Q'	Celestial Equator	
NWSE	Celestial Horizon	
ZP, NQ'	Co-Latitude of the place	
$C_{10}D_{10}$, $C_{11}D_{11}$, $C_{12}D_{12}$, C_1D_1, C_2D_2, C_3D_3	Declination for the Cusp points C_{10}, C_{11}, C_{12}, C_1, C_2 & C_3 respectively	

Item (Part)	Description	Remarks
OM	Declination of the sun when it is at (S_1) ascendant	
OXX'O	Diurnal Circle of the body (Sun)	
ϒC_{10}, ϒC_{11}, ϒC_{12}, ϒC_1, ϒC_2, ϒC_3	Ecliptic longitude for the Cusp points C_{10}, C_{11}, C_{12}, C_1, C_2 & C_3 respectively	
ϒY♎Y'	Ecliptic	
QPD_{10}, QPD_{11}, QPD_{12}, QPD_1, QPD_2, QPD_3	Hour Angle for the Cusp points C_{10}, C_{11}, C_{12}, C_1, C_2 & C_3 respectively	
QM, QPM, XPO	Hour Angle for the sun when it is at (S_1) ascendant	
PN	Latitude of the place	
$Q'D_{10}$, $Q'D_{11}$, $Q'D_{12}$, $Q'D_1$, $Q'D_2$, $Q'D_3$	Meridian Distance (Lower) for the Cusp points C_{10}, C_{11}, C_{12}, C_1, C_2 & C_3 respectively	
QD_{10}, QD_{11}, QD_{12}, QD_1, QD_2, QD_3	Meridian Distance (Upper) for the Cusp points C_{10}, C_{11}, C_{12}, C_1, C_2 & C_3 respectively	
NZSZ'	Meridian of the place	
P, P'	North & South Celestial Poles	
N, S, E, W	North, South, East & West point in the Horizon	
ϒA_{10}, ϒA_{11}, ϒA_{12}, ϒA_1, ϒA_2, ϒA_3	Oblique Ascension for the Cusp points C_{10}, C_{11}, C_{12}, C_1, C_2 & C_3 respectively	
YϒQ, O♎M	Obliquity of the ecliptic	
PC_{10}, PC_{11}, PC_{12}, PC_1, $P'C_2$, $P'C_3$	Polar Distance for the Cusp points C_{10}, C_{11}, C_{12}, C_1, C_2 & C_3 respectively	
PT_{10}, PT_{11}, PT_{12}, PT_1, $P'T_2$, $P'T_3$	Polar Height for the Cusp points C_{10}, C_{11}, C_{12}, C_1, C_2 & C_3 respectively (Arc Distance)	
ZE	Prime Vertical (Part of the Arc)	
ϒD_{10}, ϒD_{11}, ϒD_{12}, ϒD_1, ϒD_2, ϒD_3	Right Ascension for the Cusp points C_{10}, C_{11}, C_{12}, C_1, C_2 & C_3 respectively	
XO, QM	Semi-Diurnal Arc of the Sun	
X'O, Q'M	Semi-Nocturnal Arc of the sun	
ϒQ, ϒPQ	Sidereal Time of the body	
ϒ, ♎	Vernal equinox, autumnal equinox	
Z, Z'	Zenith & Nadir	
ZO	Zenith Distance of the Sun at O	

Note:

In fig. 8, some of the common points are given with more than one notation (name) for understanding purpose only. Also because of similar nature, only one side of the celestial sphere showing all the relevant points, angles, arcs etc., are described and on the other side only with few important points, angles & arcs are given.

In the placidus house system (Semi-Arc Method), basically there are four plane surfaces involves for the formulation of house cusps namely Observer's meridian (or meridian, NZSZ' in Fig. 8), Celestial horizon (NWSE in Fig. 8), Celestial Equator (ϒQ♎Q' in Fig. 8) and the Ecliptic (ϒY♎Y' in Fig. 8). For any given place the Observer's meridian (or meridian) and the celestial horizon will always bisect each other orthogonally (in Fig. 8, Angle ZGE, EGZ', ZGW, WGZ' all are equal and 90 degree, Where G is the centre of the celestial sphere). These two planes bisects the Celestial Equator into four equal parts each 90 Degree (in Fig. 8, Arc QE, EQ', Q'W, WQ all are equal to 90 Degree).

Similarly they bisect the Diurnal Circle (OXX'O in Fig. 8) of the celestial body (sun) into four parts (in Fig. 8 Arcs XO, OX' are on the eastern side of the horizon both put together covering 180 degree and the other two parts are on the western side and they are symmetric with respect to meridian). The arc XO (See Fig. 8) is above the celestial horizon and the other Arc OX' (See Fig. 8) is below the celestial horizon. The former part is known as Semi-Diurnal Arc and the later part is known as Semi-Nocturnal Arc. Each of this Arc will subtend an angle called hour Angle XPO (H in Fig. 8) and OPX' (H' in Fig. 8) at the celestial pole respectively. Similar is the case for the western side of the horizon, which is not represented in the Fig.8 due to symmetric reason.

The hour angle subtended by each Arc at the celestial pole is trisected (each part is equal to **q** and **g** for respective angles XPO = H & OPX'= H' in Fig. 8) by the declination circles/hour circles ($PC_{11}D_{11}$, $PC_{12}D_{12}$, PC_2D_2 & PD_3C_3 in Fig. 8) passing from north celestial poles which will divide the ecliptic into 12(twelve) Parts. These are all the houses of the placidus (Semi Arc) system & the intersection points are called House Cusps (C_1, C_2, C_3, C_{10}, C_{11}, and C_{12} in Fig. 8. The other cusps C_7, C_8, C_9, C_4, C_5, and C_6 are respectively opposite to the former cusps mentioned).

In astrology, the point of the ecliptic or the sign of the zodiac that rises in the eastern horizon at the time of a person's birth or any other event is called **ascendant** (1st Cusp C_1, in Fig. 8). It is nothing but the point of intersection of ecliptic with the horizon on the eastern side and the diametrically opposite point on the west side of the horizon is called **Descendent** (7th Cusp C_7).

Similarly the ecliptic will intersect the meridian at two diametrically opposite points the one that is on the upper meridian is called **Medium Coeli** (MC) or Mid Heaven. The **Mid Heaven** is the highest point in the horizon marks the tenth (10th Cusp C_{10}, in Fig. 8) house of the placidus system. The other diametrically opposite point on the lower meridian is called **Imum Coeli** (IC). It marks the fourth (4th Cusp C_4, in Fig. 8) house of the placidus system.

These four house cusps 1(one), 4(four), 7(seven) & 10(Ten) are called angular cusps and they are common for most house systems. The other remaining cusps are called non-angular cusps and in general they are different for each house systems. Each cusp will have one diametrically opposite house cusp. Hence by finding any six consecutive cusps the balance cusps that are diametrically opposite to it can be found by just either adding or subtracting 180 degree from it. First let us discuss the details of the fig. 8 shown above which is used for the formulation of the house cusp equations.

In Fig. 8,When the Sun is at O on the eastern side of the celestial horizon, its right ascension, declination, ascensional difference, oblique ascension, Semi-Diurnal Arc and Semi-Nocturnal Arc are ♈QM, MO, EM, ♈QE, QM & Q'M respectively. Angle XPO = Angle QPM. Thus both the Arc XO & QM represents the Semi-Diurnal Arc distance (in degree) of the body when it is at the horizon O. EO is the amplitude of the Sun when it is at O denoted by **a** (say).

Semi-Diurnal Arc QM = Angle QPM (=H) = QE + EM ---Eqn.(39)

Where QE is the equatorial arc between the celestial horizon and the meridian, which is always 90. EM is the ascensional difference of a body when it is on the horizon. Let us consider the spherical triangle OEM (in Fig 8, Fig 9(a)), which is right angle at M with respect to celestial equator due to the reason that the declination circle is always intersect the celestial equator at any points orthogonally. NQ' is the co-latitude of the place. Thus the spherical angle at E is equal to the co-latitude (OEM = 90 – ϕ) & Fig 9(b) is it's circular parts.

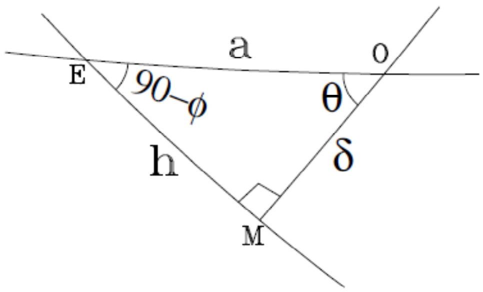

Fig.9 (a) Right Spherical Triangle OEM

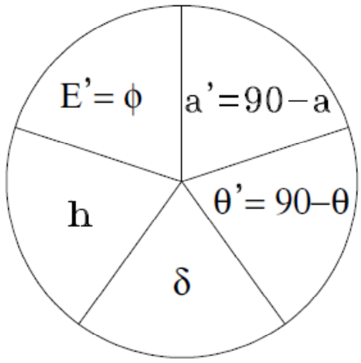

Fig.9 (b) Napier's Circular Parts

Using Rule-1 for the Middle part h,

$\sin(h) = \tan(\phi)\tan(\delta)$ ---Eqn.(40)

Using Rule-2 for the Middle part a',

$\sin(90 - a) = \cos(h)\cos(\delta)$

$\cos(a) = \cos(h)\cos(\delta)$ ---Eqn.(41)

Thus from the Eqn.40, it is very clear that the ascensional difference of a body is a function of latitude of the place and the declination of the body. Depending upon the sign of these two values, the ascensional difference will be either –ve or +ve. Now we can write the Eqn.39 as

H = 90 + h ---Eqn.(42)

Taking cos on both sides we get,

$\cos(H) = \cos(90 + h)$

$\cos(H) = -\sin(h)$ ---Eqn.(43)

Using the Eqn.40, The above equation may be written as

$\cos(H) = -\tan(\phi)\tan(\delta)$ ---Eqn.(44)

Also we have, sum of the semi-diurnal Arc & semi-nocturnal Arc is 180, i.e,

QPM + MPQ' = 180

H + H' = 180 ---Eqn.(45)

H' = 180 – H

i.e., Semi-nocturnal Arc (H')= 180 - semi-diurnal Arc (H)

Thus from the Eqn.44, it is very clear that the Semi-Diurnal Arc (or) hour angle of a body is a function of latitude of the place and the declination of a body. Alternatively the Eqn.44 can be derived from the formula used for the calculation of altitude of a celestial body in the horizontal co-ordinate system by substituting altitude = 0 when the body is on the horizon. For more details any astronomy books may be referred.

The Ascensional difference ($A_{10}D_{10}$, $A_{11}D_{11}$, $A_{12}D_{12}$, A_1D_1, A_2D_2, A_3D_3, A_4D_4 in Fig. 8) for the Cusp points C_{10}, C_{11}, C_{12}, C_1, C_2, C_3 & C_4 are the arc distance measured along the celestial equator (♈Q♎Q' in Fig.8) between their cardinal circle ($NC_{10}S$, $NC_{11}S$, $NC_{12}S$, NC_1S, NC_2S, NC_3S, NC_4S in Fig. 8) and the declination circle of the respective cusp points ($PC_{10}D_{10}$, $PC_{11}D_{11}$, $PC_{12}D_{12}$, PC_1D_1, PC_2D_2, PD_3C_3, PD_4C_4 in Fig. 8). It is nothing but the difference in celestial equatorial distance between the right ascension (♈D_{10}, ♈D_{11}, ♈D_{12}, ♈D_1, ♈D_2, ♈D_3, ♈D_4 in Fig.8) and the oblique ascension (♈A_{10}, ♈A_{11}, ♈A_{12}, ♈A_1, ♈A_2, ♈A_3, ♈A_4 in Fig.8) of the respective cusp points. When a celestial body is at the meridian (S_{10} and S_4 in Fig. 8) then the Ascensional difference of the celestial body will be equal to zero ($A_{10}D_{10}$ = 0 and A_4D_4 = 0 in Fig. 8) because both the cardinal circle and the declination circle of the celestial body will coincide with the meridian. Similarly when a celestial body is at the celestial horizon then the Ascensional difference of the celestial body will be maximum. So the Ascensional difference(h_d) for a cusp point at any Meridian distance (M_d) can be found in proportion to the Ascensional difference (h_a) divided by the Semi-Arc (S_a) distance when the body is on the celestial horizon as

$$h_d = \frac{h_a}{S_a} \times M_d \qquad \text{---Eqn.(46)}$$

From Fig. 8) we have semi-diurnal Arc & Semi-nocturnal Arc as

$$H = 3 \times q \qquad \text{---Eqn.(47)}$$

$$H' = 3 \times g \qquad \text{---Eqn.(48)}$$

Alternatively,

$$q = \frac{H}{3} \qquad \text{---Eqn.(49)}$$

$$g = \frac{H'}{3} \qquad \text{---Eqn.(50)}$$

From Eqn.47 & 48, We can rewrite the Eqn.45 as

$$3 \times q + 3 \times g = 180$$

$$q + g = 60 \qquad \text{---Eqn.(51)}$$

From Fig.8, the celestial body (Sun) at S_1 has to traverse the amount **q** (one third of the semi-diurnal Arc) in its diurnal circle XOX' to reach the position at S_2 and the declination circle passing through this position ($PS_{12}D_{12}$) will intersect the ecliptic at the cusp point C_{12}, at the same time it will cover the equatorial arc distance (A_1A_{12}) between the cardinal circles (NC_1S, $NC_{12}S$) and another amount **q** (one third of the semi-diurnal Arc) to reach the position at S_{11} and the declination circle passing through this position ($PS_{11}D_{11}$) will intersect the ecliptic at the cusp point C_{11}, at the same time it will cover the equatorial arc distance ($A_{12}A_{11}$) between the cardinal circles ($NC_{12}S$, $NC_{11}S$) and another amount **q** (one third of the semi-diurnal Arc) to reach the position at S_{10} (on the meridian) and the declination circle passing through this position ($PS_{10}D_{10}$) will intersect the ecliptic at the cusp point C_{10}, at the same time it will cover the equatorial arc distance ($A_{11}A_{10}$) between the cardinal circles ($NC_{11}S$, $NC_{10}S$) and like manner it has to traverse succession for another three more amount equal to **q** to arrive at the cusp points C_9, C_8 & C_7 respectively. Similarly it has to traverse the amount **g** (one third of the semi-nocturnal Arc) in succession for the remaining six parts to arrive from C_7, to C_1 respectively.

We can notice from the above that when the sun traverse from S_1 to S_{10} the ascensional difference for the respective position gradually decreases and becomes zero at the meridian. Thus the difference in the corresponding equatorial arc distance (A_1A_{12}, $A_{12}A_{11}$, $A_{11}A_{10}$ and so on) remains constant which is 30 degree arc. This further implies that the difference in oblique ascension (♈A_1 - ♈A_{12}, ♈A_{12} - ♈A_{11}, ♈A_{11} - ♈A_{10} and so on) between the any two consecutive cusps is always 30 degree. This further means that the each quadrant of the celestial equator (QE, EQ', Q'W & WQ) is always trisected by the cardinal circles of the cusps in the respective quadrants.

From all the above we can establish the relation between the right ascension, semi-diurnal Arc, Semi-nocturnal Arc, oblique ascension, ascensional difference as given below.

Oblique ascension (Method-1)

Since the difference in oblique ascension between the any two consecutive cusps is always 30 degree. We can compute the Oblique ascension of the cusps by simply adding 30 degree from the

RA of 10th Cusp (Right Ascension = Oblique ascension at this cusp, Since $A_{10}D_{10}$=0) for the succeeding cusps progressively as shown below.

$$A_{10} = \text{RAMC} \quad \text{---Eqn.(52)}$$

$$A_{11} = \text{RAMC} + 30 \quad \text{---Eqn.(53)}$$

$$A_{12} = \text{RAMC} + 60 \quad \text{---Eqn.(54)}$$

$$A_{1} = \text{RAMC} + 90 \quad \text{---Eqn.(55)}$$

$$A_{2} = \text{RAMC} + 120 \quad \text{---Eqn.(56)}$$

$$A_{3} = \text{RAMC} + 150 \quad \text{---Eqn.(57)}$$

Semi-Arc Distance

The Semi-Arc distances for the cusps are calculated using the Eqns. (40/42 or 44, 45, 49 & 50) in proportion to the meridian (upper) distance of the respective cusps as shown below.

$H_{10} = QD_{10}$

$$H_{10} = 0 \quad \text{---Eqn.(58)}$$

$H_{11} = QD_{11}$ (or) $H_{11} = D_{10}D_{11}$

$$H_{11} = q \quad \text{--Eqn.(59a)}$$

$$H_{11} = \frac{1}{3} \times H \quad \text{---Eqn.(59)}$$

$H_{12} = QD_{12}$ (or) $H_{12} = D_{10}D_{11} + D_{11}D_{12}$

$$H_{12} = 2 \times q \quad \text{--Eqn.(60a)}$$

$$H_{12} = \frac{2}{3} \times H \quad \text{---Eqn.(60)}$$

$H_{1} = QD_{1}$ (or) $H_{1} = D_{10}D_{11} + D_{11}D_{12} + D_{12}D_{1}$

$$H_{1} = 3 \times q \quad \text{--Eqn.(61a)}$$

$$H_{1} = H \quad \text{---Eqn.(61)}$$

$H_{2} = QD_{2}$ (or) $H_{2} = QD_{1} + D_{1}D_{2}$

$$H_{2} = 3 \times q + 1 \times g \quad \text{--Eqn.(62a)}$$

$$H_{2} = H + \frac{1}{3} \times H' \quad \text{---Eqn.(62)}$$

$H_{3} = QD_{3}$ (or) $H_{3} = QD_{1} + D_{1}D_{2} + D_{2}D_{3}$

$$H_{3} = 3 \times q + 2 \times g \quad \text{--Eqn.(63a)}$$

$$H_{3} = H + \frac{2}{3} \times H' \quad \text{---Eqn.(63)}$$

Right ascension (Method-1)

The Semi-Arc distance computed above is added with 10^{th} cusp Right ascension (♈Q) denoted RAMC (= Sidereal time in degree) to get the right ascension as shown below.

R_{10} = ♈Q

R_{11} = ♈Q + QD_{11} (or) R_{11} = ♈Q + $D_{10}D_{11}$

R_{12} = ♈Q + QD_{12} (or) R_{12} = ♈Q + $D_{10}D_{11} + D_{11}D_{12}$

R_{1} = ♈Q + QD_{1}

R_{2} = ♈Q + QD_{2} (or) R_{2} = ♈Q + $QD_{1} + D_{1}D_{2}$

R_{3} = ♈Q + QD_{3} (or) R_{3} = ♈Q + $QD_{1} + D_{1}D_{2} + D_{2}D_{3}$

Using ϒQ =RAMC & Eqns. 58 to 63, We can write the above equations as

$R_{10} = RAMC$ ---Eqn.(64)

$R_{11} = RAMC + q$ --Eqn.(65a)

$R_{11} = RAMC + \frac{1}{3}\times H$ ---Eqn.(65)

$R_{12} = RAMC + 2\times q$ --Eqn.(66a)

$R_{12} = RAMC + \frac{2}{3}\times H$ ---Eqn.(66)

$R_{1} = RAMC + 3\times q$ --Eqn.(67a)

$R_{1} = RAMC + H$ ---Eqn.(67)

$R_{2} = RAMC + 3\times q + 1\times g$ --Eqn.(68a)

$R_{2} = RAMC + H + \frac{1}{3}\times H'$ ---Eqn.(68)

$R_{3} = RAMC + 3\times q + 2\times g$ --Eqn.(69a)

$R_{3} = RAMC + H + \frac{2}{3}\times H'$ ---Eqn.(69)

Ascensional difference

The ascensional differences for the cusps are calculated using the Eqn. (46) in proportion to the meridian distance of the respective cusps as shown below.

$h_{10} = A_{10}D_{10}$

$h_{10} = 0$ ---Eqn.(70)

$h_{11} = A_{11}D_{11}$; $h_{11} = \frac{q}{3q}\times h_a$

$h_{11} = \frac{1}{3}\times h_a$ ---Eqn.(71)

$h_{12} = A_{12}D_{12}$; $h_{12} = \frac{2q}{3q}\times h_a$

$h_{12} = \frac{2}{3}\times h_a$ ---Eqn.(72)

$h_{1} = A_{1}D_{1}$; $h_{1} = \frac{3q}{3q}\times h_a$

$h_{1} = h_a$ ---Eqn.(73)

$h_{2} = A_{2}D_{2}$; $h_{2} = \frac{2g}{3g}\times h_a$

$h_{2} = \frac{2}{3}\times h_a$ ---Eqn.(74)

$h_{3} = A_{3}D_{3}$; $h_{3} = \frac{g}{3g}\times h_a$

$h_{3} = \frac{1}{3}\times h_a$ ---Eqn.(75)

Note: Where h_a can be calculated using Eqn.40

Oblique ascension (Method-2)

The ascensional difference calculated in the Eqns.(70 to 75) are deducted from respective cusp Right ascension computed in Method-1 to get the oblique ascension as shown below.

$A_{10} = \Upsilon D_{10} - A_{10}D_{10}$; $A_{10} = R_{10} - h_{10}$

$A_{10} = R_{10}$ ---Eqn.(76)

$A_{11} = \Upsilon D_{11} - A_{11}D_{11}$; $A_{11} = R_{11} - h_{11}$

$A_{11} = R_{11} - \frac{1}{3} \times h_a$ ---Eqn.(77)

$A_{12} = \Upsilon D_{12} - A_{12}D_{12}$; $A_{12} = R_{12} - h_{12}$

$A_{12} = R_{12} - \frac{2}{3} \times h_a$ ---Eqn.(78)

$A_1 = \Upsilon D_1 - A_1D_1$; $A_1 = R_1 - h_1$

$A_1 = R_1 - h_a$ ---Eqn.(79)

$A_2 = \Upsilon D_2 - A_2D_2$; $A_2 = R_2 - h_2$

$A_2 = R_2 - \frac{2}{3} \times h_a$ ---Eqn.(80)

$A_3 = \Upsilon D_3 - A_3D_3$; $A_3 = R_3 - h_3$

$A_3 = R_3 - \frac{1}{3} \times h_a$ ---Eqn.(81)

Right ascension (Method-2)

The ascensional difference calculated in the Eqns.(70 to 75) are added with respective cusp oblique ascension computed in Method-1 to get the Right ascension as shown below.

$R_{10} = A_{10} + A_{10}D_{10}$; $R_{10} = A_{10} + h_{10}$

$R_{10} = A_{10}$ ---Eqn.(82)

$R_{11} = A_{11} + A_{11}D_{11}$; $R_{11} = A_{11} + h_{11}$

$R_{11} = A_{11} + \frac{1}{3} \times h_a$ ---Eqn.(83)

$R_{12} = A_{12} + A_{12}D_{12}$; $R_{12} = A_{12} + h_{12}$

$R_{12} = A_{12} + \frac{2}{3} \times h_a$ ---Eqn.(84)

$R_1 = A_1 + A_1D_1$; $R_1 = A_1 + h_1$

$R_1 = A_1 + h_a$ ---Eqn.(85)

$R_2 = A_2 + A_2D_2$; $R_2 = A_2 + h_2$

$R_2 = A_2 + \frac{2}{3} \times h_a$ ---Eqn.(86)

$R_3 = A_3 + A_3D_3$; $R_3 = A_3 + h_3$

$R_3 = A_3 + \frac{1}{3} \times h_a$ ---Eqn.(87)

Now we have computed almost all the relevant relations that may be used for the formulation of house cusp equations. But unfortunately from Eqn. 28 & 32 (refer Summary of Ecliptic longitude Equations) it is evident that either declination or the Right ascension of the celestial body is required to compute the ecliptic longitude of the cusps. Again the Right ascension is the function of declination as mentioned in Method 1 & 2 discussed above. So we need the declination of the respective cusp to calculate the ecliptic longitude, which is unknown at this initial stage, thus some trial and error method or any other solution of equations may be applied to arrive it. Basically this is involved for the computation of non-angular houses.

Formulation of equation for the Ascendant

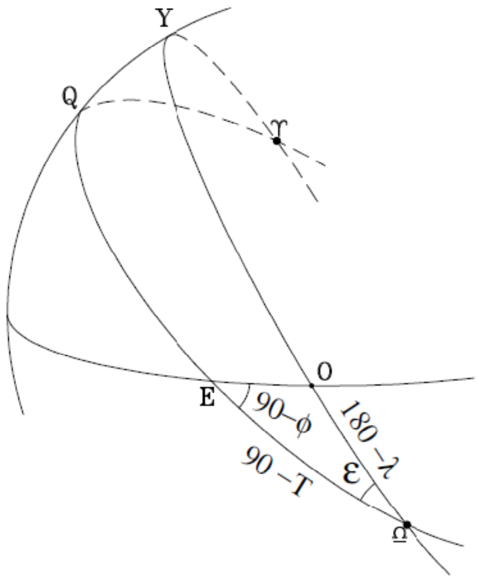

Fig.10) Spherical Triangle OE☊

In Fig. 10, all the notations used are same as Fig. 8). In Spherical Triangle OE☊ Let,

♈O = ♈YO = **λ** (Ecliptic longitude of the celestial body at O ie., at Ascendant)

♈Q = **T** (Sidereal time in Degrees for the given Date, Time & Place)

QE = 90 (Quadrant part of the Celestial equator)

O☊E = **ε** (Obliquity of the ecliptic)

OE☊ = 90 – **φ** (in Fig.8, NQ' is the co-latitude of the place)

♈Q☊ = 180 (as ♈, ☊ are the diametrically opposite points in the celestial equator)

♈Y☊ = 180 (as ♈, ☊ are the diametrically opposite points in the ecliptic)

☊E = ♈Q☊ – ♈Q – QE

☊E = 180 – T – 90

☊E = 90 – T

☊O = ♈Y☊ – ♈YO

☊O = 180 – **λ**

From the Spherical Triangle OE☊ (in Fig.10), It is very clear that the problem is related with four consecutive parts of the spherical triangle in which all the three parts are known except one side O☊. Hence cotangent Rule may be applied to solve the problem as explained below.

Outer Angle (OA) = 90 – **φ** (= OE☊)

Inner Angle (IA) = **ε** (=O☊E)

Outer Side (OS) = 180 – **λ** (=O☊)

Inner Side (IS) = 90 – T (= E☊)

We have the cotangent Rule as,

$$\cos(IS)\cos(IA) = \cot(OS)\ \sin(IS) - \cot(OA)\sin(IA)$$

Substituting the parameters in the above eqn., We get,

$$\cos(90-T)\ \cos(\varepsilon) = \cot(180-\lambda)\ \sin(90-T) - \cot(90-\phi)\sin(\varepsilon)$$

$$\sin(T)\cos(\varepsilon) = -\cot(\lambda)\cos(T) - \tan(\phi)\sin(\varepsilon)$$

$$-\cot(\lambda)\cos(T) = \sin(T)\cos(\varepsilon) + \tan(\phi)\sin(\varepsilon)$$

$$\cot(\lambda) = \frac{\sin(T)\cos(\varepsilon) + \tan(\phi)\sin(\varepsilon)}{-\cos(T)} \quad \text{--Eqn.(88a)}$$

$$\tan(\lambda) = \frac{-\cos(T)}{\sin(T)\cos(\varepsilon) + \tan(\phi)\sin(\varepsilon)} \quad \text{---Eqn.(88)}$$

Substituting $T = RAMC$ in the above Eqns. 88a & 88, We get,

$$\cot(\lambda) = \frac{\sin(RAMC)\cos(\varepsilon) + \tan(\phi)\sin(\varepsilon)}{-\cos(RAMC)} \quad \text{--Eqn.(89a)}$$

$$\tan(\lambda) = \frac{-\cos(RAMC)}{\sin(RAMC)\cos(\varepsilon) + \tan(\phi)\sin(\varepsilon)} \quad \text{---Eqn.(89)}$$

In astrology, once we know the sidereal time (RAMC) for the given date and time of birth (or any event) place, Geographical (or Geocentric) latitude (**ϕ**) of the birth (or any event) place & the obliquity of the ecliptic (**ε**), we can compute the ecliptic longitude (λ) of the ascendant by directly substituting it in the above eqn. 89a or 89.

Formulation of equation for the Medium Coeli (MC) or Mid Heaven

From fig.10), considering the part of the spherical triangle ♈QY as shown below (in fig.11a). In fig. 11(a), all the notations used are same as fig. 8) & fig. 11(b) shows the circular parts of the right angled Spherical Triangle ♈QY. Let,

♈Y = **λ** (Ecliptic longitude of the Cusp at Meridian i.e., at Medium Coeli (MC))
♈Q = **T** (Sidereal time in Degrees for the given Date, Time & Place)
QY = **δ** (Declination of the Cusp at Medium Coeli (MC))
Y♈Q = **ε** (Obliquity of the ecliptic)
♈YQ = **θ** (Corner angle between the ecliptic and the declination circle)
♈QY = **90** (as celestial equator and declination circle is always interest orthogonally)

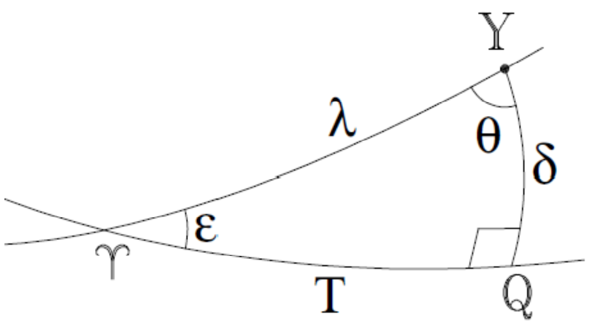

Fig.11 (a) Right Spherical Triangle ♈QY

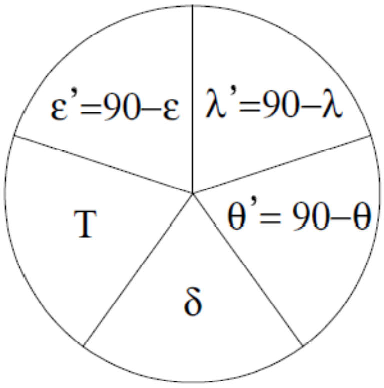

Fig.11 (b) Napier's Circular Parts

Using Rule-1 for the Middle part **ε'**,

$$\sin(90-\varepsilon) = \tan(T)\tan(90-\lambda)$$

$$\cos(\varepsilon) = \tan(T)\cot(\lambda)$$

$$\tan(\lambda) = \frac{\tan(T)}{\cos(\varepsilon)} \qquad \text{--Eqn.(90a)}$$

Substituting $T = RAMC$ in the above Eqns. 90a, we get,

$$\tan(\lambda) = \frac{\tan(RAMC)}{\cos(\varepsilon)} \qquad \text{---Eqn.(90)}$$

In astrology, once we know the sidereal time (RAMC) for the given date and time of birth (or any event) place & the obliquity of the ecliptic (**ε**), we can compute the ecliptic longitude (λ) of the Mid Heaven by directly substituting in the above eqn. 90. From the Eqn. 90) it is very clear that the Medium Coeli (MC) is not a function of Geographical (or Geocentric) latitude (**ϕ**) of the birth (or any event) place thus for all the places in the same longitude will have the same ecliptic longitude (λ) at Mid Heaven (Cusp C_{10}) or Medium Coeli (MC).

Formulation of equations for the non-Angular Cusps

From fig.8), a typical non-angular Cusp and its associated spherical triangles (♈AC, ADC & CPT) are considered in fig.12 as shown below. All the notations used here are same as fig. 8) except removing the suffix number in the notations wherever required/appropriate to derive a general-purpose equations applicable for all the non-angular houses. The below Fig.12) is a typical celestial sphere view looking below from the top of the observer's Head is shown. The readers may refer fig.8) (east side view) to correlate with this below top view for better understanding. For the reader's understanding the typical notations used in fig.12 are once again described here below.

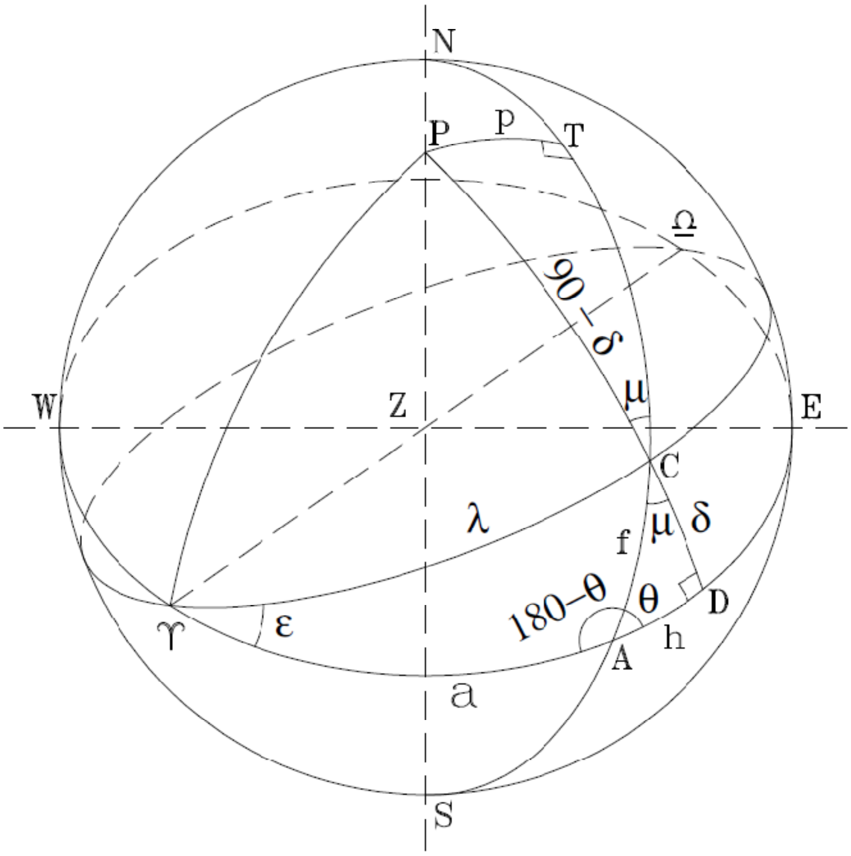

Fig.12) Celestial Sphere (Top View) & Spherical Triangles ♈AC, ADC & CPT

Description of the Notations:

Item (Part)	Description	Remarks
AD	Ascensional difference for the Cusp point C	
NCS	Cardinal Circle for the Cusp point C	
PT	Cardinal Pole for the Cusp point C	
♈ADE	Celestial Equator	
NWSE	Celestial Horizon	
CD	Declination for the Cusp point C	
♈C	Ecliptic longitude for the Cusp point C	
♈C♎	Ecliptic	
NPZS	Meridian of the place	
P	North Celestial Pole	
N, S, E, W	North, South, East & West point in the Horizon	
♈A	Oblique Ascension for the Cusp point C	
C♈D	Obliquity of the ecliptic	
PC	Polar Distance for the Cusp point **C**	
PT	Pole Height for the Cusp point C (Arc Distance)	
♈D	Right Ascension for the Cusp point C	
♈, ♎	Vernal equinox, autumnal equinox	
Z	Zenith	

From fig.12), Consider the part of the right-angled spherical triangle ACD as shown below (in fig.13a), all the notations used are same as fig. 12). The Fig. 13(b) shows the circular parts of the right-angled Spherical Triangle ACD. Let,

AD = **h** (Ascensional difference for the Cusp point C)
CD = **δ** (Declination of the Cusp point C)
ACD = **μ** (Corner angle between the declination circle and the cardinal circle)
CAD = **θ** (Corner angle between the celestial equator and the cardinal circle)
ADC = **90** (as celestial equator and declination circle is always interest orthogonally)
AC = **f** (side of the spherical triangle considered)

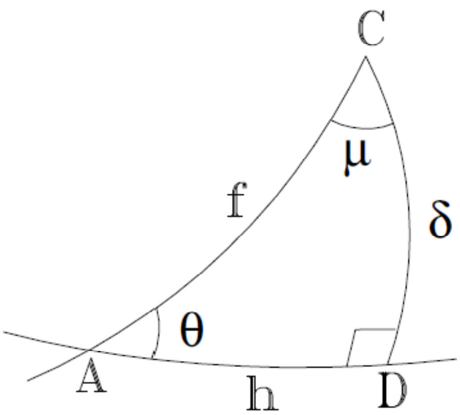

Fig.13 (a) Right Spherical Triangle ACD

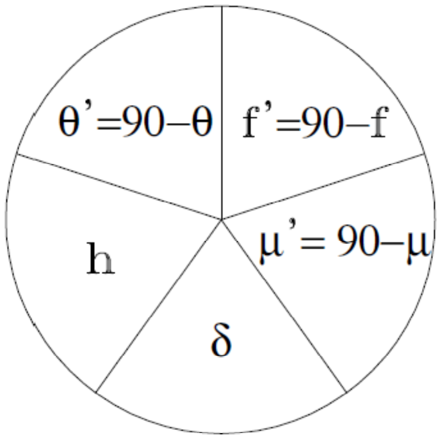

Fig.13 (b) Napier's Circular Parts

Using Rule-2 for the Middle part **f '**,

$\sin(90 - f) = \cos(h)\cos(\delta)$

$\cos(f) = \cos(h)\cos(\delta)$ --Eqn.(91)

Using Rule-2 for the Middle part **h**,

$\sin(h) = \cos(90 - f)\cos(90 - \mu)$

$$\sin(h) = \sin(f)\sin(\mu) \qquad \text{---Eqn.(92)}$$

From Pythagorean theorem, we have,

$$\sin(f) = \sqrt{1-\cos^2(f)} \qquad \text{---Eqn.(93)}$$

Substituting values from Eqn.(91) in Eqn.(93) we get,

$$\sin(f) = \sqrt{1-\cos^2(h)\cos^2(\delta)} \qquad \text{---Eqn.(94)}$$

Substituting values from Eqn.(94) in Eqn.(92) we get,

$$\sin(\mu) = \frac{\sin(h)}{\sqrt{1-\cos^2(h)\cos^2(\delta)}} \qquad \text{---Eqn.(95)}$$

Using Rule-2 for the Middle part **θ'**,

$$\sin(90 - \theta) = \cos(90-\mu)\cos(\delta)$$

$$\cos(\theta) = \sin(\mu)\cos(\delta) \qquad \text{---Eqn.(96)}$$

Substituting values from Eqn.(95) in Eqn.(96) we get,

$$\cos(\theta) = \frac{\sin(h)\cos(\delta)}{\sqrt{1-\cos^2(h)\cos^2(\delta)}} \qquad \text{---Eqn.(97)}$$

In Eqn.(97) & Eqn.(100) right hand side values are equal, Thus we get,

$$\cos(\theta) = \sin(p)$$

We have $\cos(90 - p) = \sin(p)$, So we get from the above equation,

$$\theta = (90 - p) \qquad \text{---Eqn.(98)}$$

From fig.12), Consider the part of the right-angled spherical triangle CPT as shown below (in fig.14a), all the notations used are same as fig. 12). The Fig. 14(b) shows the circular parts of the right-angled Spherical Triangle CPT. Let,

CT = **r** (side of the spherical triangle considered)
PT = **p** (Pole Height or Polar Elevation of the Cusp point C)
PCT = **μ** (Corner angle between the declination circle and the cardinal circle)
CPT = **η** (Corner angle between the declination circle and the cardinal Pole)
CTP = **90** (by definition, cardinal Pole is taken as perpendicular to cardinal circle)
CP = 90 - **δ** (Polar distance of the Cusp point C)

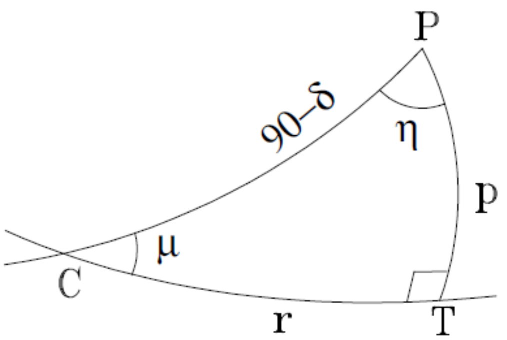

Fig.14 (a) Right Spherical Triangle ACD

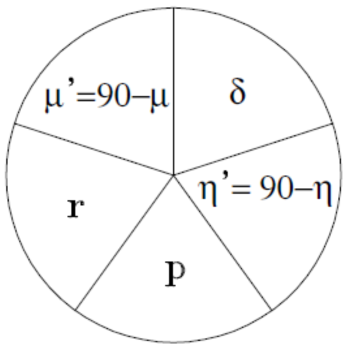

Fig.14 (b) Napier's Circular Parts

Using Rule-2 for the Middle part **p**,

$$\sin(p) = \cos(90-\mu)\cos(\delta)$$

$$\sin(p) = \sin(\mu)\cos(\delta)$$

$$\sin(\mu) = \frac{\sin(p)}{\cos(\delta)} \qquad \text{--Eqn.(99)}$$

Equating Eqn. (99) & (95), We get

$$\frac{\sin(p)}{\cos(\delta)} = \frac{\sin(h)}{\sqrt{1-\cos^2(h)\cos^2(\delta)}}$$

$$\sin(p) = \frac{\sin(h)\cos(\delta)}{\sqrt{1-\cos^2(h)\cos^2(\delta)}} \qquad \text{--Eqn.(100)}$$

From Pythagorean theorem, we have,

$$\cos(p) = \sqrt{1-\sin^2(p)} \qquad \text{--Eqn.(101)}$$

Substituting values from Eqn.(100) in Eqn.(101) we get,

$$\cos(p) = \sqrt{1 - \frac{\sin^2(h)\cos^2(\delta)}{1-\cos^2(h)\cos^2(\delta)}}$$

$$\cos(p) = \sqrt{\frac{1-\cos^2(h)\cos^2(\delta) - \sin^2(h)\cos^2(\delta)}{1-\cos^2(h)\cos^2(\delta)}}$$

$$\cos(p) = \sqrt{\frac{1-\cos^2(\delta)(\cos^2(h)+\sin^2(h))}{1-\cos^2(h)\cos^2(\delta)}}$$

From Pythagorean theorem, we have,

$\cos^2(h)+\sin^2(h)=1$, So we get above equation as,

$$\cos(p) = \sqrt{\frac{1-\cos^2(\delta)}{1-\cos^2(h)\cos^2(\delta)}}$$

From Pythagorean theorem, we have,

$\sin^2(\delta)=1-\cos^2(\delta)$, So we get above equation as,

$$\cos(p) = \sqrt{\frac{\sin^2(\delta)}{1-\cos^2(h)\cos^2(\delta)}}$$

$$\cos(p) = \frac{\sin(\delta)}{\sqrt{1-\cos^2(h)\cos^2(\delta)}} \qquad \text{--Eqn.(102)}$$

Dividing the two Eqn.(100) & Eqn.(102) on either side, we get,

$$\tan(p) = \frac{\sin(h)\cos(\delta)}{\sin(\delta)}$$

$$\tan(p) = \frac{\sin(h)}{\tan(\delta)} \qquad \text{--Eqn.(103)}$$

From fig.12), Consider the part of the spherical triangle ♈AC as shown below (in fig.15), all the notations used are same as fig. 12).

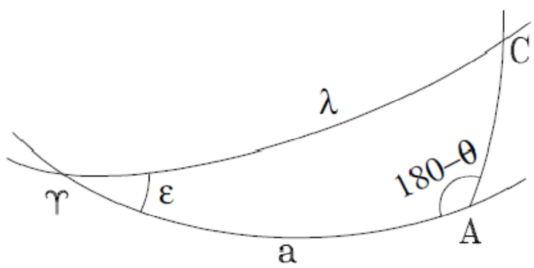

Fig.15 Spherical triangle ♈AC

In Spherical Triangle ♈AC Let,

♈C = **λ** (Ecliptic longitude of the cusp considered)

♈A = **a** (oblique ascension of the cusp considered)

C♈A = **ε** (Obliquity of the ecliptic)

♈AC = 180 - **θ**

From the Spherical Triangle ♈AC (in Fig.15), It is very clear that the problem is related with four consecutive parts of the spherical triangle in which all the three parts are known except one side ♈C. Hence cotangent Rule may be applied to solve the problem as explained below.

Outer Angle (OA) = 180 – **θ** (= ♈AC)

Inner Angle (IA) = **ε** (= C♈A)

Outer Side (OS) = **λ** (= ♈C)

Inner Side (IS) = **a** (= ♈A)

We have the cotangent Rule as,

$$\cos(IS)\cos(IA) = \cot(OS)\ \sin(IS) - \cot(OA)\sin(IA)$$

Substituting the parameters in the above eqn., We get,

$$\cos(a)\ \cos(\varepsilon) = \cot(\lambda)\ \sin(a) - \cot(180 - \theta)\sin(\varepsilon)$$
$$\cos(\mathrm{a})\cos(\varepsilon) = \cot(\lambda)\sin(a) + \cot(\theta)\sin(\varepsilon)$$
$$\cot(\lambda)\sin(a) = -\cot(\theta)\sin(\varepsilon) + \cos(\mathrm{a})\cos(\varepsilon)$$

Substituting values from Eqn.(98) in the above equation, we get,

$$\cot(\lambda)\sin(a) = -\cot(90 - p)\sin(\varepsilon) + \cos(a)\cos(\varepsilon)$$
$$\cot(\lambda)\sin(a) = \cos(a)\cos(\varepsilon) - \tan(p)\sin(\varepsilon)$$
$$\cot(\lambda) = \frac{\cos(a)\cos(\varepsilon) - \tan(p)\sin(\varepsilon)}{\sin(a)}$$
$$\tan(\lambda) = \frac{\sin(a)}{\cos(a)\cos(\varepsilon) - \tan(p)\sin(\varepsilon)} \qquad \text{--Eqn.(104)}$$
$$\tan(\lambda) = \frac{\sin(a)\cos(p)}{\cos(p)\cos(a)\cos(\varepsilon) - \sin(p)\sin(\varepsilon)} \qquad \text{--Eqn.(105)}$$

The above Eqn.105 may be written in another form by using the following auxiliary components. Let,

$$\mathrm{N}\sin(\mathrm{M}) = \sin(p) \qquad \text{--Eqn.(106)}$$
$$\mathrm{N}\cos(\mathrm{M}) = \cos(p)\cos(a) \qquad \text{--Eqn.(107)}$$

Using the values from Equation 106, 107 in Eqn. 105, we get,

$$\tan(\lambda) = \frac{\sin(a)\cos(p)}{\mathrm{N}\cos(M)\cos(\varepsilon) - \mathrm{N}\sin(M)\sin(\varepsilon)}$$
$$\tan(\lambda) = \frac{\sin(a)\cos(p)}{\mathrm{N}\cos(M + \varepsilon)} \qquad \text{--Eqn.(108)}$$

From Eqn.107, We get,

$$\cos(p) = \frac{\mathrm{N}\cos(M)}{\cos(a)} \qquad \text{--Eqn.(109)}$$

Using the values from Equation 109 in Eqn. 108, we get,

$$\tan(\lambda) = \frac{\mathrm{N}\cos(M)\sin(a)}{\mathrm{N}\cos(\mathrm{M} + \varepsilon)\cos(a)}$$

$$\tan(\lambda) = \frac{\cos(M)\tan(a)}{\cos(M+\varepsilon)} \qquad \text{--Eqn.(110)}$$

Dividing Eqn.(106) by Eqn.(107) we get,

$$\tan(M) = \frac{\tan(p)}{\cos(a)} \qquad \text{--Eqn.(111)}$$

Using the values from Eqn.(103) in Eqn.111, We get,

$$\tan(M) = \frac{\sin(h)}{\tan(\delta)\cos(a)} \qquad \text{--Eqn.(112)}$$

As mentioned earlier, many years ago may be a century ago, the values are computed by using the logarithmic tables for which the equations 110 & 112 are found more convenient. But nowadays we have scientific calculators/computers, which will do the job very fast.

From the Eqn. 110 & 112) it very clear that the ecliptic longitude (λ) of the non-angular cusps is a function of obliquity of the ecliptic (ε), Ascensional difference, declination and oblique ascension of the cusps. But again Ascensional difference is a function of declination, Geographical (or Geocentric) latitude (ϕ) of the birth (or any event) place and oblique ascension is a function of sidereal time (RAMC) for the given date and time of birth (or any event) place. So we need the declination of the respective cusp to calculate the ecliptic longitude, which is unknown at this initial stage, thus some trial and error method or any other solution of equations may be applied to arrive it.

Now we have derived all the necessary equations for the calculation of house cusps and the following section gives the detailed step-by-step procedure for the same.

Step-by-step procedure for computation of house cusps

Inputs

The data available for the computation of house cusps are date, time and Place (latitude & longitude) of birth (or any event) & obliquity of the ecliptic.

Step-1

Compute the siderial time (i.e., RAMC) for the given birth (or any event) date, time and place for which a separate literature may be referred. Once the RAMC is calculated then compute the angular cusps ascendant (C_1) and Mid Heaven (C_{10}) position using the direct equations 89 & 90 respectively.

$$C_1 = Tan^{-1}\left(\frac{-\cos(RAMC)}{\sin(RAMC)\cos(\varepsilon)+\tan(\phi)\sin(\varepsilon)}\right)$$

$$C_{10} = Tan^{-1}\left(\frac{\tan(RAMC)}{\cos(\varepsilon)}\right)$$

Now we need to calculate the non-angular cusps as follows.

Step-2

Compute the oblique ascension for the non-angular cusps using the direct equations 53, 54, 56 & 57 respectively.

$$A_{11} = \text{RAMC}+30$$

$$A_{12} = \text{RAMC}+60$$

$$A_2 = \text{RAMC}+120$$

$$A_3 = \text{RAMC}+150$$

Step-3

For calculating position of the non-angular cusps we need many parameters in which the one of the main parameter is the declination. Unfortunately we do not have the values at this initial stage. So to start with we need to assume some approximate values for the same. For the best approximation we can assume that the ascensional difference for all the non-angular cusps is zero. Thus the oblique ascension of the non-angular cusps is equal to the right ascension. Now using the declination equation 25 (refer summary of declination equation) we can compute the approximate declination for the respective non-angular cusps.

$$\mathrm{D}_{11} = Tan^{-1}(\sin(\mathrm{A}_{11})\tan(\varepsilon))$$
$$\mathrm{D}_{12} = Tan^{-1}(\sin(\mathrm{A}_{12})\tan(\varepsilon))$$
$$\mathrm{D}_{2} = Tan^{-1}(\sin(\mathrm{A}_{2})\tan(\varepsilon))$$
$$\mathrm{D}_{3} = Tan^{-1}(\sin(\mathrm{A}_{3})\tan(\varepsilon))$$

Step-4

Compute the ascensional difference for the non-angular cusps for which we need two parameters the one is the latitude of the place and the other one is declination. From the basic inputs we know the latitude and the declination from step-3. Now using the equations 71, 72, 74 & 75 we can compute the ascensional difference for the respective non-angular cusps.

$$\mathrm{h}_{11} = \frac{1}{3} \times Sin^{\ 1}(\tan(D_{11})\tan(\phi))$$
$$\mathrm{h}_{12} = \frac{2}{3} \times Sin^{-1}(\tan(D_{12})\tan(\phi))$$
$$\mathrm{h}_{2} = \frac{2}{3} \times Sin^{-1}(\tan(D_{2})\tan(\phi))$$
$$\mathrm{h}_{3} = \frac{1}{3} \times Sin^{-1}(\tan(D_{3})\tan(\phi))$$

Step-5

Now the ecliptic longitude of the non-angular cusps can be calculated either by using direct equation 103 & 105 or by auxiliary equations 110 & 112. Here auxiliary equations are used for the computation of ecliptic longitude of the non-angular cusps. For this we have to find the auxiliary component that is required for ecliptic longitude calculation. This auxiliary component is the function of ascensional difference, declination and oblique ascension of the non-angular cusps. Now using the equation 112 we can compute the auxiliary component for the respective non-angular cusps by substituting the values arrived from Step-2, 3 & 4 above.

$$\mathrm{M}_{11} = Tan^{-1}\left(\frac{\sin(h_{11})}{\tan(D_{11})\cos(A_{11})}\right)$$
$$\mathrm{M}_{12} = Tan^{-1}\left(\frac{\sin(h_{12})}{\tan(D_{12})\cos(A_{12})}\right)$$
$$\mathrm{M}_{2} = Tan^{-1}\left(\frac{\sin(h_{2})}{\tan(D_{2})\cos(A_{2})}\right)$$
$$\mathrm{M}_{3} = Tan^{-1}\left(\frac{\sin(h_{3})}{\tan(D_{3})\cos(A_{3})}\right)$$

Step-6

The ecliptic longitude of the non-angular cusps is a function of auxiliary component, oblique ascension & obliquity of the ecliptic. Using the equation 110 we can compute the ecliptic longitude for the respective non-angular cusps by substituting the values arrived from Step-2 & 5 above.

$$C_{11} = Tan^{-1}\left(\frac{\cos(M_{11})\tan(A_{11})}{\cos(M_{11}+\varepsilon)}\right)$$

$$C_{12} = Tan^{-1}\left(\frac{\cos(M_{12})\tan(A_{12})}{\cos(M_{12}+\varepsilon)}\right)$$

$$C_{2} = Tan^{-1}\left(\frac{\cos(M_{2})\tan(A_{2})}{\cos(M_{2}+\varepsilon)}\right)$$

$$C_{3} = Tan^{-1}\left(\frac{\cos(M_{3})\tan(A_{3})}{\cos(M_{3}+\varepsilon)}\right)$$

Step-7

The ecliptic longitudes of the non-angular cusps calculated are based on the assumed approximate declination. So we have to compute the actual declination of the respective cusps for convergence. Now using the declination equation 22 (refer summary of declination equation) we can compute the actual declination (G's) for the respective non-angular cusps as shown below.

$$G_{11} = Sin^{-1}(\sin(C_{11})\sin(\varepsilon))$$
$$G_{12} = Sin^{-1}(\sin(C_{12})\sin(\varepsilon))$$
$$G_{2} = Sin^{-1}(\sin(C_{2})\sin(\varepsilon))$$
$$G_{3} = Sin^{-1}(\sin(C_{3})\sin(\varepsilon))$$

Substitute, $D_{11} = G_{11}$; $D_{12} = G_{12}$; $D_2 = G_2$; and $D_3 = G_3$. And repeat the steps from 4 to 7 until the difference in recent declination (G's) and previously calculated declination (D's) is very minimum. At this stage the values for C's are the ecliptic longitudes for the required non-angular cusps.

The readers may note that the Article (Ref. [1]) on "An Astrological House Formulary" by Michael P. Munkasey, shows a step under the heading 'The Placidian House System Formulation' where the ecliptic longitude of the cusp is substituted for the declination instead of substituting the actual declination (a missing step) which is wrong and it will not give the correct results.

Step-8

The other opposite cusps can be calculated by either adding or subtracting 180 degree from the respective cusp values. Add 180 if the longitude is less than 180 and subtract 180 if the longitude is greater than or equal to 180.

$$C_4 = C_{10} \pm 180$$
$$C_5 = C_{11} \pm 180$$
$$C_6 = C_{12} \pm 180$$
$$C_7 = C_1 \pm 180$$
$$C_8 = C_2 \pm 180$$
$$C_9 = C_3 \pm 180$$

Summary & Conclusion

The Author is very happy to share the above information with readers and believes that this will be an eye opener for those who seek for the mathematics, trigonometry and astronomy part of the house cusp equations. Once the fundamental is very clear then anyone can solve this type of spherical astronomy problems very easily.

The Author believes that the most of the readers may straightaway look into the trigonometry equations that are provided in this article for computing the house cusps. However it is advised for the readers those who want to compute the house cusps without understanding the astronomy, trigonometry and its basic principles, will have tough time and may get lot of pain to arrive at the correct values. The steps/methods described here may be one of the ways to solve the problem. Therefore try to understand well and one can play around the problem for solving it. GOOD LUCK!!

References

1. An article on "An Astrological House Formulary" by Michael P. Munkasey
2. The Progressed Horoscope by Alan Leo
3. The Spherical Basis for Astrology by Joseph G. Dalton

Editor's Note

The above article is 'one of its kinds'. It has been put under Research Perspective because the author has taken laborious literature survey, mathematical exposition and erudite explanation of the fundamentals of house divisions that serve as the basics of applied astrology. This article is worthy of being included as a module in any astrological course/training to impart the knowledge of the main premises of zodiacal divisions of houses. JASA highly appreciates Mr. D. Senthilathiban's contribution.

This article titled "**Trigonometry Behind The House Cusps**" by ***D.Senthilathiban*** is a part of the astrology Journal published in the Journal for Advancement of Stellar Astrology (JASA) volume-1, Issue 2, Sept-Oct 2011, Printed Pages 43-77.

The Editor's web page address/URL http://www.astroandrewkp.org/journal.php